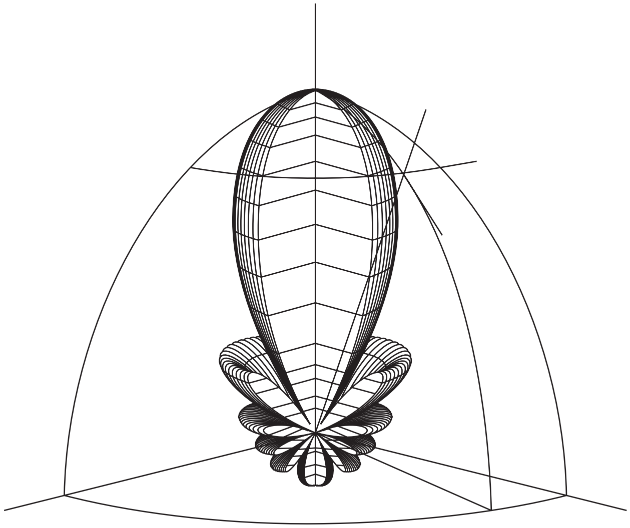
CHAPTER 2

# Antenna Basics



## 2–1 INTRODUCTION

Welcome to the wonderful world of antennas, its language and culture; to the aperture family (effective and scattering), the lobe family (main, side, back, and grating); to beamwidths, directivity, and gain.

Antennas are three-dimensional and live in a world of beam area, steradians, square degrees, and solid angle. Antennas have impedances (self and mutual). They couple to all of space and have a temperature measured in kelvins. Antennas have polarizations: linear, elliptical, and circular.

This chapter will make you fluent in the language of antennas and comfortable in its culture. The topics of this chapter include:

- Basic parameters
- Patterns
- Beam area
- Beam efficiency
- Directivity and gain
- Physical and effective apertures
- Scattering aperture and radar cross section
- The radio link (Friis formula)
- Apertures of dipoles and $\lambda/2$ antennas
- Radiation resistance
- Antenna impedance
- Antenna duality
- Sources of radiation
- Field zones
- Shape-impedance considerations
- Polarization

## 2–2 BASIC ANTENNA PARAMETERS

A *radio antenna* may be defined as the structure associated with the region of transition between a guided wave and a free-space wave, or vice versa. Antennas convert electrons to photons, or vice versa.[1]

Regardless of antenna type, all involve the same basic principle that radiation is produced by accelerated (or decelerated) charge. The *basic equation of radiation* may be expressed simply as

$$\boxed{\dot{I}L = Q\dot{v} \qquad (\text{A m s}^{-1}) \quad \textbf{\textit{Basic radiation equation}}} \tag{1}$$

where

$\dot{I}$ = time-changing current, A $s^{-1}$
$L$ = length of current element, m
$Q$ = charge, C
$\dot{v}$ = time change of velocity which equals the acceleration of the charge, m $s^{-2}$
$L$ = length of current element, m

Thus, ***time-changing current radiates and accelerated charge radiates.*** For steady-state harmonic variation, we usually focus on current. For transients or pulses, we focus on charge.[2] The radiation is perpendicular to the acceleration, and the radiated power is proportional to the square of $\dot{I}L$ or $Q\dot{v}$.

The two-wire transmission line in Fig. 2–1*a* is connected to a radio-frequency generator (or transmitter). Along the uniform part of the line, energy is guided as a plane Transverse ElectroMagnetic Mode (TEM) wave with little loss. The spacing between wires is assumed to be a small fraction of a wavelength. Further on, the transmission line opens out in a tapered transition. As the separation approaches the order of a wavelength or more, the wave tends to be radiated so that the opened-out line acts like an antenna which launches a free-space wave. The currents on the transmission line flow out on the antenna and end there, but the fields associated with them keep on going.

The transmitting antenna in Fig. 2–1*a* is a region of transition from a guided wave on a transmission line to a free-space wave. The receiving antenna (Fig. 2–1*b*) is a region of transition from a space wave to a guided wave on a transmission line. Thus, ***an antenna is a transition device, or transducer, between a guided wave and a free-space wave, or vice-versa.*** The antenna is a device which interfaces a circuit and space.

From the circuit point of view, the antennas appear to the transmission lines as a resistance $R_r$, called the *radiation resistance*. It is not related to any resistance in the antenna itself but is a resistance coupled from space to the antenna terminals.

In the transmitting case, the radiated power is absorbed by objects at a distance: trees, buildings, the ground, the sky, and other antennas. In the receiving case, passive radiation from distant objects or active radiation from other antennas raises the apparent temperature

[1] A photon is a quantum unit of electromagnetic energy equal to $hf$, where $h$ = Planck's constant (= $6.63 \times 10^{-34}$ J s) and $f$ = frequency (Hz).

[2] A pulse radiates with a broad bandwidth (the shorter the pulse the broader the bandwidth). A sinusoidal variation results in a narrow bandwidth (theoretically zero at the frequency of the sinusoid if it continues indefinitely).

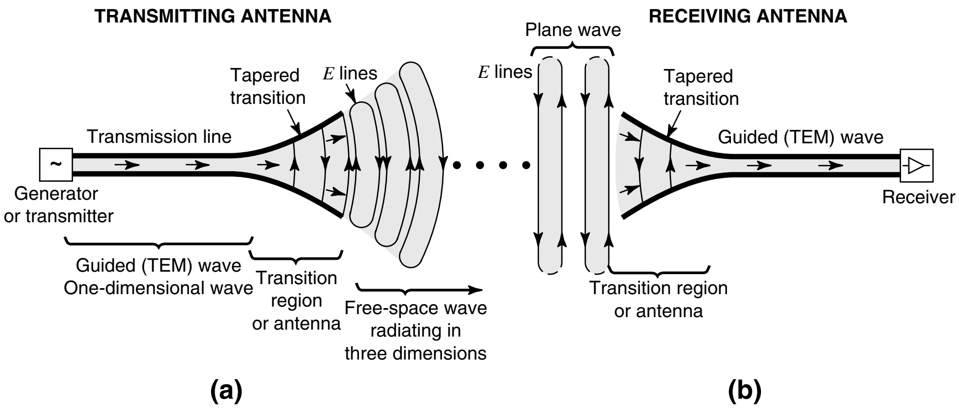



**Figure 2–1**
(*a*) Radio (or wireless) communication link with transmitting antenna and (*b*) receiving antenna. The receiving antenna is remote from the transmitting antenna so that the spherical wave radiated by the transmitting antenna arrives as an essentially plane wave at the receiving antenna.

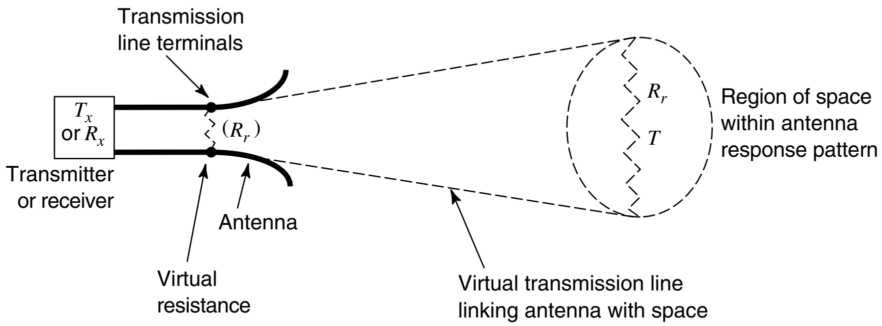



**Figure 2–2**
Schematic representation of region of space at temperature *T* linked via a virtual transmission line to an antenna.

of $R_r$. For lossless antennas this temperature has nothing to do with the physical temperature of the antenna itself but is related to the temperature of distant objects that the antenna is "looking at," as suggested in Fig. 2–2. In this sense, a receiving antenna (and its associated receiver) may be regarded as a remote-sensing temperature-measuring device.

As pictured schematically in Fig. 2–2, the radiation resistance $R_r$ may be thought of as a "virtual" resistance that does not exist physically but is a quantity coupling the antenna to distant regions of space via a "virtual" transmission line.[1]

[1]It is to be noted that the radiation resistance, the antenna temperature, and the radiation patterns are functions of the frequency. In general, the patterns are also functions of the distance at which they are measured, but at distances which are large compared to the size of the antenna and large compared to the wavelength, the pattern is independent of distance. Usually the patterns of interest are for this far-field condition.

## 2–3 PATTERNS

Both the radiation resistance $R_r$, and its temperature $T_A$ are simple scalar quantities. The radiation patterns, on the other hand, are three-dimensional quantities involving the variation of field or power (proportional to the field squared) as a function of the spherical coordinates $\theta$ and $\phi$. Figure 2–3 shows a three-dimensional field pattern with pattern radius $r$ (from origin to pattern boundary at the dot) proportional to the field intensity in the direction $\theta$ and $\phi$. The pattern has its *main lobe* (maximum radiation) in the $z$ direction ($\theta = 0$) with *minor lobes* (side and back) in other directions.

To completely specify the radiation pattern with respect to field intensity and polarization requires three patterns:

1. The $\theta$ component of the electric field as a function of the angles $\theta$ and $\phi$ or $E_\theta(\theta, \phi)$ ($\mathrm{V\ m^{-1}}$) as in Figs. 2–3 and 2–4.

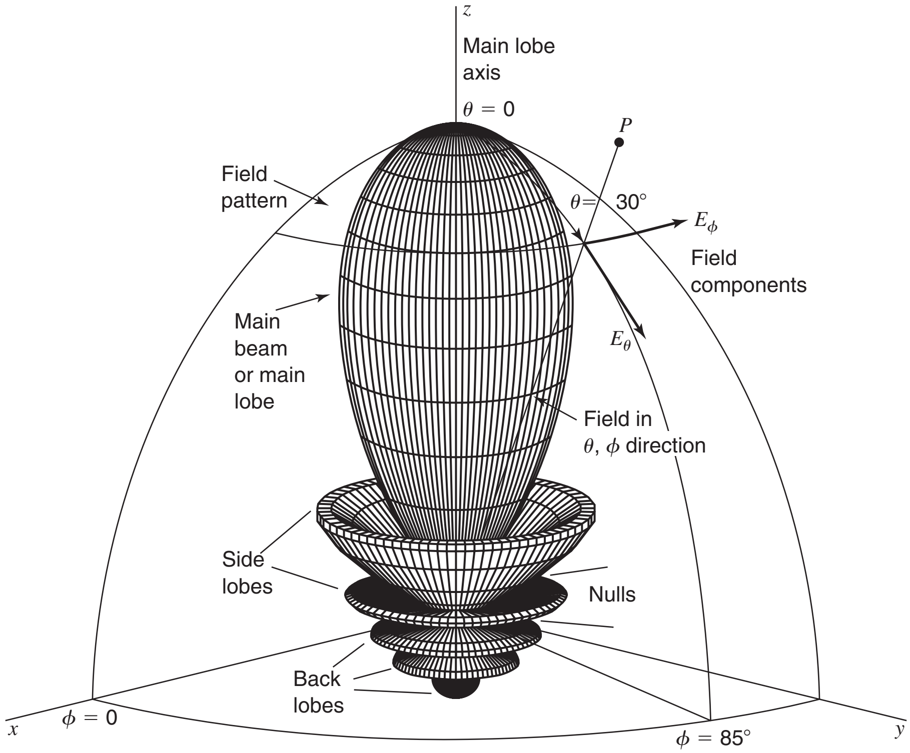



**Figure 2–3**
Three-dimensional field pattern of a directional antenna with maximum radiation in $z$-direction at $\theta = 0^\circ$. Most of the radiation is contained in a *main beam* (or *lobe*) accompanied by radiation also in *minor lobes* (*side* and *back*). Between the lobes are *nulls* where the field goes to zero. The radiation in any direction is specified by the angles $\theta$ and $\phi$. The direction of the point $P$ is at the angles $\theta = 30^\circ$ and $\phi = 85^\circ$. This pattern is symmetrical in $\phi$ and a function only of $\theta$.

**2.** The $\phi$ component of the electric field as a function of the angles $\theta$ and $\phi$ or $E_\phi(\theta, \phi)$ (V m$^{-1}$).

**3.** The phases of these fields as a function of the angles $\theta$ and $\phi$ or $\delta_\theta(\theta, \phi)$ and $\delta_\phi(\theta, \phi)$ (rad or deg).

Any field pattern can be presented in three-dimensional spherical coordinates, as in Fig. 2–3, or by plane cuts through the main-lobe axis. Two such cuts at right angles, called the *principal plane patterns* (as in the $xz$ and $yz$ planes in Fig. 2–3) may be required but if the pattern is symmetrical around the $z$ axis, one cut is sufficient.

Figures 2–4*a* and 2–4*b* are principal plane field and power patterns in polar coordinates. The same pattern is presented in Fig. 2–4*c* in rectangular coordinates on a logarithmic, or decibel, scale which gives the minor lobe levels in more detail.

The angular beamwidth at the half-power level or *half-power beamwidth* (HPBW) (or −3-dB beamwidth) and the *beamwidth between first nulls* (FNBW) as shown in Fig. 2–4, are important pattern parameters.

Dividing a field component by its maximum value, we obtain a *normalized or relative field pattern* which is a dimensionless number with maximum value of unity. Thus, the normalized field pattern (Fig. 2–4*a*) for the electric field is given by

$$\boxed{\textbf{\textit{Normalized field pattern}} = E_\theta(\theta, \phi)_n = \frac{E_\theta(\theta, \phi)}{E_\theta(\theta, \phi)_{\max}} \qquad \text{(dimensionless)}} \tag{1}$$

The half-power level occurs at those angles $\theta$ and $\phi$ for which $E_\theta(\theta, \phi)_n = 1/\sqrt{2} = 0.707$.

At distances that are large compared to the size of the antenna and large compared to the wavelength, the shape of the field pattern is independent of distance. Usually the patterns of interest are for this *far-field* condition.

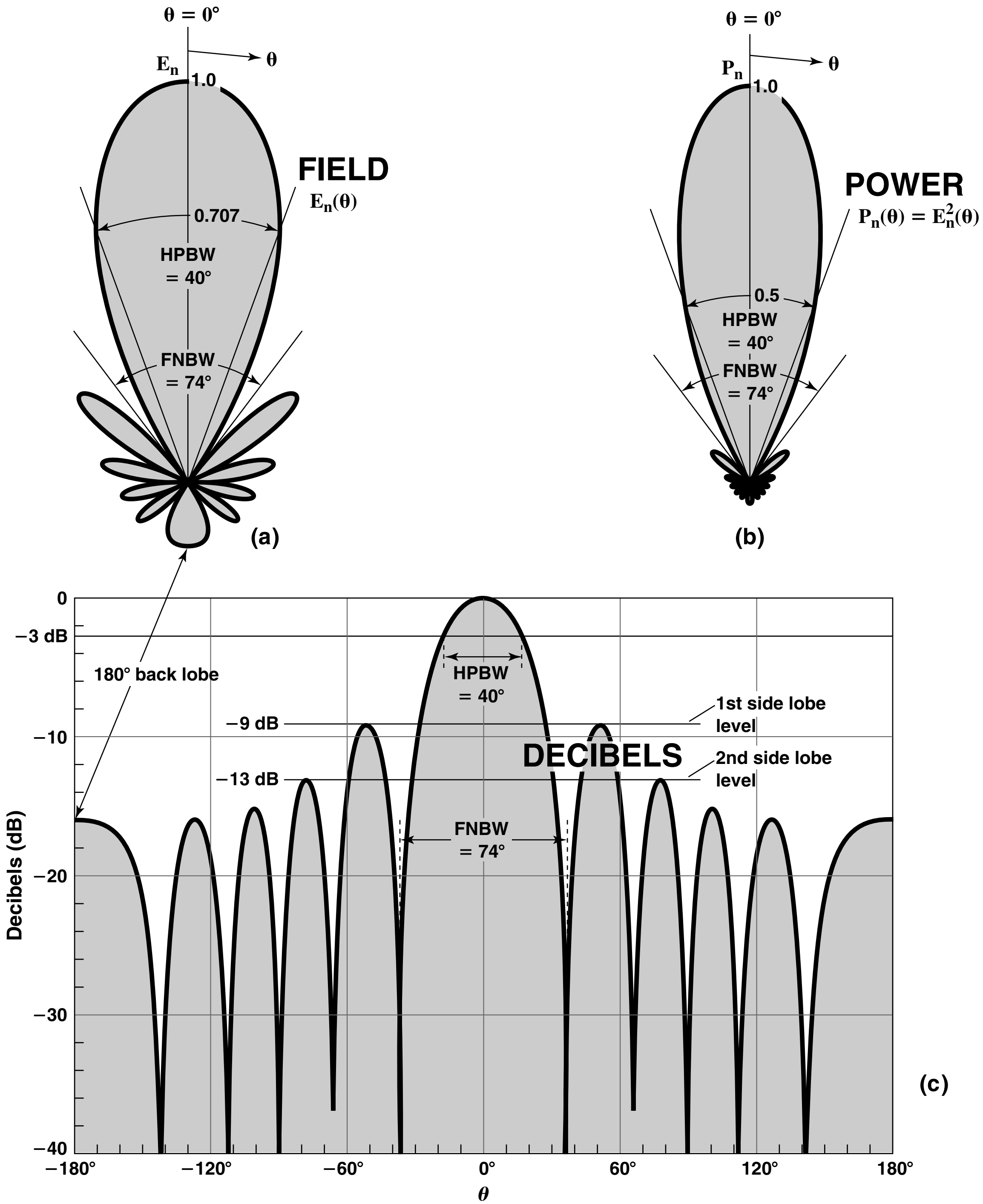
Patterns may also be expressed in terms of the *power per unit area* [or Poynting vector $S(\theta, \phi)$].[1] Normalizing this power with respect to its maximum value yields a *normalized power pattern* as a function of angle which is a dimensionless number with a maximum value of unity. Thus, the normalized power pattern (Fig. 2–4*b*) is given by

$$\boxed{\textbf{\textit{Normalized power pattern}} = P_n(\theta, \phi)_n = \frac{S(\theta, \phi)}{S(\theta, \phi)_{\max}} \qquad \text{(dimensionless)}} \tag{2}$$

where

$S(\theta, \phi)$ = Poynting vector = $\left[E_\theta^2(\theta, \phi) + E_\phi^2(\theta, \phi)\right] / Z_0$, W m$^{-2}$
$S(\theta, \phi)_{\max}$ = maximum value of $S(\theta, \phi)$, W m$^{-2}$
$Z_0$ = intrinsic impedance of space = 376.7 Ω

[1]Although the Poynting vector, as the name implies, is a vector (with magnitude and direction), we use here its magnitude; its direction is the far field is radially outward.


θ = 0°
$E_n$
θ
1.0
FIELD
$E_n(\theta)$
0.707
HPBW = 40°
FNBW = 74°
(a)
θ = 0°
$P_n$
θ
1.0
POWER
$P_n(\theta) = E_n^2(\theta)$
0.5
HPBW = 40°
FNBW = 74°
(b)
0
−3 dB
180° back lobe
HPBW = 40°
1st side lobe level
−9 dB
−10
DECIBELS
2nd side lobe level
−13 dB
FNBW = 74°
−20
−30
−40
Decibels (dB)
(c)
−180° −120° −60° 0° 60° 120° 180°
θ

The decibel level is given by

$$\text{dB} = 10\log_{10} P_n(\theta, \phi) \tag{3}$$

where $P_n(\theta, \phi)$ is as given by (2).

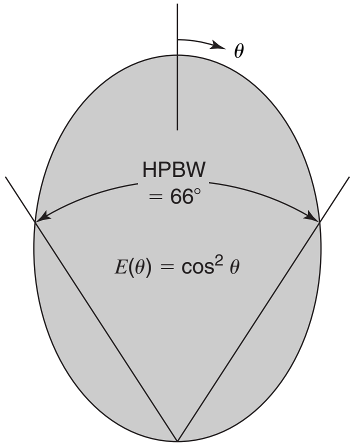
**EXAMPLE**

### 2–3.1 Half-Power Beamwidth

An antenna has a field pattern given by

$$E(\theta) = \cos^2\theta \quad \text{for } 0^\circ \leq \theta \leq 90^\circ$$

Find the half-power beamwidth (HPBW).




**Solution**

$E(\theta)$ at half power $= 0.707$. Thus $0.707 = \cos^2\theta$ so $\cos\theta = \sqrt{0.707}$ and $\theta = 33^\circ$

$$\text{HPBW} = 2\theta = 66^\circ \qquad \textit{Ans.}$$

**Figure 2–4**

Two-dimensional field, power and decibel plots of the 3-D antenna pattern of Fig. 2–3. Taking a slice through the middle of the 3-dimensional pattern of Figure 2–3 results in the 2-dimensional pattern at (*a*). It is a field pattern (proportional to the electric field $E$ in V/m) with normalized relative field $E_n(\theta) = 1$ at $\theta = 0^\circ$. The half-power beam width (HPBW) $= 40^\circ$ is measured at the $E = 0.707$ level.

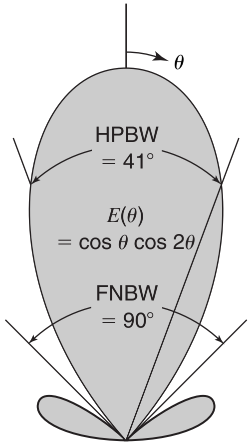
The pattern at (*b*) is a power plot of (*a*) (proportional to $E^2$) with relative power $P_n = 1$ at $\theta = 0^\circ$ and with HPBW $= 40^\circ$ as before and measured at the $P_n = 0.5$ level.

A decibel (dB) plot of (*a*) is shown at (*c*) with HPBW $= 40^\circ$ as before and measured at the −3 dB level. The first side lobes are shown at the −9 dB and second side lobes at −13 dB. Decibel plots are useful for showing minor lobe levels.

**EXAMPLE**

### 2–3.2 Half-Power Beamwidth and First Null Beamwidth

An antenna has a field pattern given by $E(\theta) = \cos\theta\cos 2\theta$ for $0^\circ \leq \theta \leq 90^\circ$. Find (*a*) the half-power beamwidth (HPBW) and (*b*) the beamwidth between first nulls (FNBW).




**Solution**

(*a*) $E(\theta)$ at half power $= 0.707$. Thus $0.707 = \cos\theta\cos 2\theta = 1/\sqrt{2}$.

$$\cos 2\theta = \frac{1}{\sqrt{2}\cos\theta} \qquad 2\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta}\right) \qquad \text{and}$$

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta'}\right)$$

Iterating with $\theta' = 0$ as a first guess, $\theta = 22.5^\circ$. Setting $\theta' = 22.5^\circ$, $\theta = 20.03^\circ$, etc., until after next iteration $\theta = \theta' = 20.47^\circ \cong 20.5^\circ$ and

$$\text{HPBW} = 2\theta = 41^\circ \qquad \textit{Ans. (a)}$$

(*b*) $0 = \cos\theta\cos 2\theta$, so $\theta = 45^\circ$ and

$$\text{FNBW} = 2\theta = 90^\circ \qquad \textit{Ans. (b)}$$

Although the radiation pattern characteristics of an antenna involve three-dimensional vector fields for a full representation, several simple single-valued scalar quantities can provide the information required for many engineering applications. These are:

- Half-power beamwidth, HPBW
- Beam area, $\Omega_A$
- Beam efficiency, $\varepsilon_M$

- Directivity $D$ or gain $G$
- Effective aperture $A_e$

The half-power beamwidth was discussed above. The others follow.

## 2–4 BEAM AREA (OR BEAM SOLID ANGLE) $\Omega_A$

In polar two-dimensional coordinates an incremental area $dA$ on the surface of a sphere is the product of the length $r\,d\theta$ in the $\theta$ direction (latitude) and $r \sin\theta\, d\phi$ in the $\phi$ direction (longitude), as shown in Fig. 2–5.

Thus,

$$dA = (r\,d\theta)(r \sin\theta\, d\phi) = r^2\, d\Omega \qquad (1)$$

where

$d\Omega$ = *solid angle* expressed in steradians (sr) or square degrees ($^\square$)

$d\Omega$ = solid angle subtended by the area $dA$

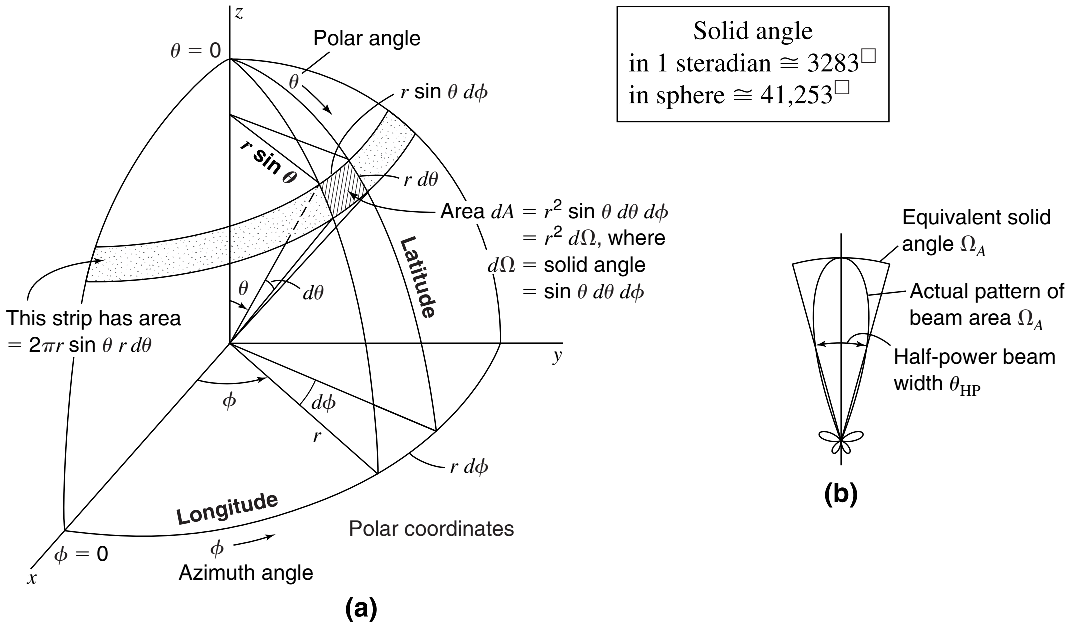



**Figure 2–5**
Polar coordinates showing incremental solid angle $dA = r^2\, d\Omega$ on the surface of a sphere of radius $r$ where $d\Omega$ = solid angle subtended by the area $dA$. ($b$) Antenna power pattern and its equivalent solid angle or beam area $\Omega_A$.

The area of the strip of width $r\,d\theta$ extending around the sphere at a constant angle $\theta$ is given by $(2\pi r \sin\theta)(r\,d\theta)$. Integrating this for $\theta$ values from 0 to $\pi$ yields the area of the sphere. Thus,

$$\text{Area of sphere} = 2\pi r^2 \int_0^{\pi} \sin\theta\, d\theta = 2\pi r^2 [-\cos\theta]_0^{\pi} = 4\pi r^2 \tag{2}$$

where $4\pi$ = solid angle subtended by a sphere, sr

Thus,

$$\begin{aligned} 1 \text{ steradian} = 1 \text{ sr} &= (\text{solid angle of sphere})/(4\pi) \\ &= 1 \text{ rad}^2 = \left(\frac{180}{\pi}\right)^2 (\text{deg}^2) = 3282.8064 \text{ square degrees} \end{aligned} \tag{3}$$

Therefore,

$$\begin{aligned} 4\pi \text{ steradians} &= 3282.8064 \times 4\pi = 41{,}252.96 \cong 41{,}253 \text{ square degrees} = 41{,}253^{\square} \\ &= \text{solid angle in a sphere} \end{aligned} \tag{4}$$

The beam area or *beam solid angle* or $\Omega_A$ of an antenna (Fig. 2–5*b*) is given by the integral of the normalized power pattern over a sphere ($4\pi$ sr)

$$\Omega_A = \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} P_n(\theta, \phi) \sin\theta\, d\theta\, d\phi \tag{5a}$$

and

$$\boxed{\Omega_A = \iint_{4\pi} P_n(\theta, \phi)\, d\Omega \qquad (\text{sr}) \qquad \textbf{\textit{Beam area}}} \tag{5b}$$

where $d\Omega = \sin\theta\, d\theta\, d\phi$, sr.

The beam area $\Omega_A$ is the solid angle through which all of the power radiated by the antenna would stream if $P(\theta, \phi)$ maintained its maximum value over $\Omega_A$ and was zero elsewhere. Thus the power radiated = $P(\theta, \phi)\Omega_A$ watts.

The *beam area* of an antenna can often be described *approximately* in terms of the angles subtended by the *half-power points* of the main lobe in the two principal planes. Thus,

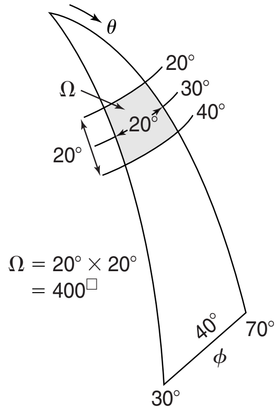
$$\boxed{\textbf{\textit{Beam area}} \cong \Omega_A \cong \theta_{HP}\phi_{HP} \qquad (\text{sr})} \tag{6}$$

where $\theta_{HP}$ and $\phi_{HP}$ are the *half-power beamwidths* (HPBW) in the two principal planes, minor lobes being neglected.

**EXAMPLE**

### 2–4.1 Solid Angle of Area in Square Degrees

Find the number of square degrees in the solid angle $\Omega$ on a spherical surface that is between $\theta = 20°$ and $\theta = 40°$ (or 70° and 50° north latitude) and between $\phi = 30°$ and $\phi = 70°$ (30° and 70° east longitude).




**Solution**

From (1)

$$\Omega = \int_{30°}^{70°} d\phi \int_{20°}^{40°} \sin\theta \, d\theta = \frac{40}{360} 2\pi [-\cos\theta]_{20}^{40}$$

$$= 0.222\pi \times 0.173 = 0.121 \text{ steradians} \qquad \text{(sr)}$$

$$= 0.121 \times 3283 = 397 \text{ square degrees} = 397^{\square} \qquad \textit{Ans.}$$

The solid angle $\Omega$ shown in the sketch may be *approximated* as the product of two angles $\Delta\theta = 20°$ and $\Delta\phi = 40° \sin 30° = 40° \times 0.5 = 20°$ where 30° is the median $\theta$ value of latitude. Thus, $\Omega = \Delta\theta \, \Delta\phi = 20° \times 20° = 400^{\square}$, which is within 3/4% of the answer given above.

**EXAMPLE**

### 2–4.2 Beam Area $\Omega_A$ of Antenna with $\cos^2\theta$ Pattern

An antenna has a field pattern given by $E(\theta) = \cos^2\theta$ for $0° \le \theta \le 90°$. This is the same pattern of Example 2–3.1. Find the beam area of this pattern.

**Solution**

From (5)

$$\Omega_A = \int_0^{2\pi} \int_0^{\pi} \cos^4\theta \sin\theta \, d\theta \, d\phi$$

$$= -2\pi \left[ \frac{1}{25} \cos^5\theta \right]_0^{\pi/2} = \frac{2\pi}{5} = 1.26 \text{ sr} \qquad \textit{Ans.}$$

From (6) an *approximate* relation for the beam area

$$\Omega_A \cong \theta_{HP}\phi_{HP} \qquad \text{(sr)}$$

where $\theta_{HP}$ and $\phi_{HP}$ are the *half-power beamwidths* (HPBW) in the two principal planes. From Example 2–3.1, $\theta_{HP} = \phi_{HP} = 66^\circ$, so

$$\Omega_A \cong \theta_{HP}\phi_{HP} = 66^2 = 4356 \text{ sq deg} = 4356^\square$$

From (3), one square radian = 3283 sq deg so

$$\text{Beam area } \Omega_A = 4356/3282 = 1.33 \text{ sr} \qquad \textit{Approx. Ans.}$$

a difference of 6%.

## 2–5 RADIATION INTENSITY

The power radiated from an antenna per unit solid angle is called the *radiation intensity U* (watts per steradian or per square degree). The normalized power pattern of the previous section can also be expressed in terms of this parameter as the ratio of the radiation intensity $U(\theta, \phi)$, as a function of angle, to its maximum value. Thus,

$$P_n(\theta, \phi) = \frac{U(\theta, \phi)}{U(\theta, \phi)_{max}} = \frac{S(\theta, \phi)}{S(\theta, \phi)_{max}} \tag{1}$$

Whereas the Poynting vector $S$ depends on the distance from the antenna (varying inversely as the square of the distance), the radiation intensity $U$ is independent of the distance, assuming in both cases that we are in the far field of the antenna (see Sec. 2–13).

## 2–6 BEAM EFFICIENCY

The (total) *beam area* $\Omega_A$ (or *beam solid angle*) consists of the main beam area (or solid angle) $\Omega_M$ plus the minor-lobe area (or solid angle) $\Omega_m$.[1] Thus,

$$\Omega_A = \Omega_M + \Omega_m \tag{1}$$

The ratio of the main beam area to the (total) beam area is called the (main) *beam efficiency* $\varepsilon_M$. Thus,

$$\boxed{\textbf{Beam efficiency} = \varepsilon_M = \frac{\Omega_M}{\Omega_A} \qquad \text{(dimensionless)}} \tag{2}$$

The ratio of the minor-lobe area ($\Omega_m$) to the (total) beam area *is* called the *stray factor*. Thus,

$$\varepsilon_m = \frac{\Omega_m}{\Omega_A} = \text{stray factor} \tag{3}$$

It follows that

$$\varepsilon_M + \varepsilon_m = 1 \tag{4}$$

[1] If the main beam is not bounded by a deep null, its extent becomes an arbitrary act of judgment.

## 2–7 DIRECTIVITY *D* AND GAIN *G*

The directivity $D$ and the gain $G$ are probably the most important parameters of an antenna.

The *directivity* of an antenna is equal to the ratio of the maximum power density $P(\theta, \phi)_{\max}$ (watts/m$^2$) to its average value over a sphere as observed in the far field of an antenna. Thus,

$$D = \frac{P(\theta, \phi)_{\max}}{P(\theta, \phi)_{\text{av}}} \qquad \textit{\textbf{Directivity from pattern}} \tag{1}$$

The directivity is a dimensionless ratio $\geq 1$.

The average power density over a sphere is given by

$$\begin{aligned} P(\theta, \phi)_{\text{av}} &= \frac{1}{4\pi} \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} P(\theta, \phi) \sin\theta \, d\theta \, d\phi \\ &= \frac{1}{4\pi} \iint_{4\pi} P(\theta, \phi) \, d\Omega \qquad (\text{W sr}^{-1}) \end{aligned} \tag{2}$$

Therefore, the directivity

$$D = \frac{P(\theta, \phi)_{\max}}{(1/4\pi) \iint_{4\pi} P(\theta, \phi) \, d\Omega} = \frac{1}{(1/4\pi) \iint_{4\pi} [P(\theta, \phi)/P(\theta, \phi)_{\max}] \, d\Omega} \tag{3}$$

and

$$D = \frac{4\pi}{\iint_{4\pi} P_n(\theta, \phi) \, d\Omega} = \frac{4\pi}{\Omega_A} \qquad \textit{\textbf{Directivity from beam area}}\ \boldsymbol{\Omega_A} \tag{4}$$

where $P_n(\theta, \phi) \, d\Omega = P(\theta, \phi)/P(\theta, \phi)_{\max}$ = normalized power pattern

Thus, the directivity is the ratio of the area of a sphere ($4\pi$ sr) to the beam area $\Omega_A$ of the antenna (Fig. 2–5*b*).

The smaller the beam area, the larger the directivity $D$. For an antenna that radiates over only half a sphere the beam area $\Omega_A = 2\pi$ sr (Fig. 2–6) and the directivity is

$$D = \frac{4\pi}{2\pi} = 2 \qquad (= 3.01 \text{ dBi}) \tag{5}$$

where dBi = decibels over isotropic

Note that the idealized *isotropic antenna* ($\Omega_A = 4\pi$ sr) has the lowest possible directivity $D = 1$. All actual antennas have directivities greater than 1 ($D > 1$). The simple short dipole has a beam area $\Omega_A = 2.67\pi$ sr and a directivity $D = 1.5$ (= 1.76 dBi).

The *gain G* of an antenna is an actual or realized quantity which is less than the directivity $D$ due to ohmic losses in the antenna or its radome (if it is enclosed). In transmitting, these losses involve power fed to the antenna which is not radiated but heats the antenna structure. A mismatch in feeding the antenna can also reduce the gain. The ratio of the gain

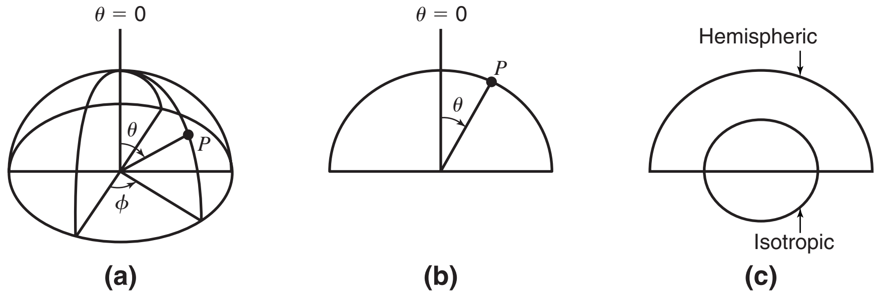



**Figure 2–6**
Hemispheric power patterns, (*a*) and (*b*), and comparison with isotropic pattern (*c*).

to the directivity is the *antenna efficiency factor*. Thus,

$$G = kD \tag{6}$$

where $k$ = efficiency factor ($0 \le k \le 1$), dimensionless.

In many well-designed antennas, $k$ may be close to unity. In practice, $G$ is always less than $D$, with $D$ its maximum idealized value.

Gain can be measured by comparing the maximum power density of the Antenna Under Test (AUT) with a reference antenna of known gain, such as a short dipole. Thus,

$$\text{Gain} = G = \frac{P_{\max}(\text{AUT})}{P_{\max}(\text{ref. ant.})} \times G(\text{ref. ant.}) \tag{7}$$

If the half-power beamwidths of an antenna are known, its directivity

$$D = \frac{41{,}253^{\square}}{\theta^{\circ}_{\text{HP}}\phi^{\circ}_{\text{HP}}} \tag{8}$$

where

$41{,}253^{\square}$ = number of square degrees in sphere = $4\pi(180/n)^2$ square degrees ($^{\square}$)
$\theta^{\circ}_{\text{HP}}$ = half-power beamwidth in one principal plane
$\phi^{\circ}_{\text{HP}}$ = half-power beamwidth in other principal plane

Since (8) neglects minor lobes, a better approximation is a

$$\boxed{D = \frac{40{,}000^{\square}}{\theta^{\circ}_{\text{HP}}\phi^{\circ}_{\text{HP}}} \qquad \textit{\textbf{Approximate directivity}}} \tag{9}$$

If the antenna has a main half-power beamwidth (HPBW) = 20° in both principal planes, its directivity

$$D = \frac{40{,}000^{\square}}{400^{\square}} = 100 \text{ or } 20\,\text{dBi} \tag{10}$$

which means that the antenna radiates 100 times the power in the direction of the main beam as a nondirectional, isotropic antenna.

The *directivity-beamwidth product* $40{,}000^{\square}$ is a rough approximation. For certain types of antennas other values may be more accurate, as discussed in later chapters.

If an antenna has a main lobe with both half-power beamwidths (HPBWs) = 20°, its directivity from (8) is *approximately*

$$D = \frac{4\pi\,(\text{sr})}{\Omega_A\,(\text{sr})} \cong \frac{41{,}253(\text{deg}^2)}{\theta^\circ_{HP}\phi^\circ_{HP}} = \frac{41{,}253(\text{deg}^2)}{20^\circ \times 20^\circ}$$

$$\cong 103 \cong 20 \text{ dBi (dB above isotropic)}$$

which means that the antenna radiates a power in the direction of the main-lobe maximum which is about 100 times as much as would be radiated by a nondirectional (isotropic) antenna for the same power input.

**EXAMPLE**

### 2–7.1 Gain of Directional Antenna with Three-Dimensional Field Pattern of Fig. 2–3

The antenna is a lossless end-fire array of 10 isotropic point sources spaced $\lambda/4$ and operating with increased directivity. See Sec. 5–6. The normalized field pattern (see Fig. 2–4*a*) is

$$E_n = \sin\left(\frac{\pi}{2n}\right)\frac{\sin(n\psi/2)}{\sin(\psi/2)} \tag{11}$$

where

$$\psi = d_r(\cos\phi - 1) - \frac{\pi}{n}$$

$$d_r = \pi/2$$

$$n = 10$$

Since the antenna is lossless, gain = directivity.

(*a*) Calculate the gain *G*.

(*b*) Calculate the gain from the approximate equation (9).

(*c*) What is the difference?

**Solution**

(*a*) From (4)

$$\text{Gain } G = \frac{4\pi}{\displaystyle\iint_{4\pi} P_n(\theta,\phi)\,d\Omega}\,d\theta\,d\phi \tag{12}$$

where $P_n(\theta,\phi)$ = normalized power pattern = $[E_n(\theta,\phi)]^2$.
Introducing the given parameters into (11) and (12),

$$G = 17.8 \text{ or } 12.5 \text{ dB} \qquad \textit{Ans. (a)}$$

(*b*) From (9) and HPBW = 40° in Fig. 2–4,

$$\text{Gain} = \frac{40{,}000^\square}{(40^\circ)^2} = 25 \text{ or } 14 \text{ dB} \qquad \textit{Ans. (b)}$$

(*c*) $\Delta G = 25/17.8 = 1.40$ or 1.5 dB    *Ans. (c)*

The difference is mostly due to the large minor lobes of the pattern. Changing the formula to

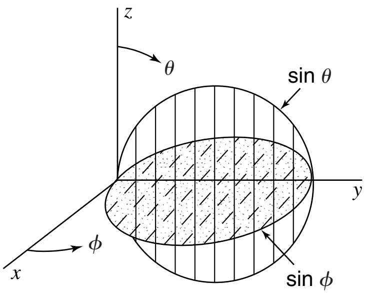
$$\text{Gain} = \frac{28{,}000^{\square}}{(40^\circ)^2} = 17.5 \text{ or } 12.4 \text{ dB}$$

the gain is much closer to that in (*a*). This approximation is considered more appropriate for end-fire arrays with increased directivity.

**EXAMPLE**

### 2–7.2 Directivity

The normalized field pattern of an antenna is given by $E_n = \sin\theta \sin\phi$, where $\theta$ = zenith angle (measured from $z$ axis) and $\phi$ = azimuth angle (measured from $x$ axis) (see figure). $E_n$ has a value only for $0 \le \theta \le \pi$ and $0 \le \phi \le \pi$ and is zero elsewhere (pattern is unidirectional with maximum in $+y$ direction). Find (*a*) the exact directivity, (*b*) the approximate directivity from (8), and (*c*) the decibel difference.




Unidirectional $\sin\theta$ and $\sin\phi$ field patterns.

**Solution**

$$D = \frac{4\pi}{\int_0^\pi \int_0^\pi \sin^3\theta \sin^2\phi \, d\theta \, d\phi} = \frac{4\pi}{2\pi/3} = 6 \qquad \textit{Ans. (a)}$$

$$D \cong \frac{41{,}253^{\square}}{90^\circ \times 90^\circ} = 5.1 \qquad \textit{Ans. (b)}$$

$$10 \log \frac{6.0}{5.1} = 0.7 \text{ dB} \qquad \textit{Ans. (c)}$$

## 2–8 DIRECTIVITY AND RESOLUTION

The resolution of an antenna may be defined as equal to half the beamwidth between first nulls (FNBW)/2,[1] for example, an antenna whose pattern FNBW = 2° has a resolution of 1° and, accordingly, should be able to distinguish between transmitters on two adjacent

[1]Often called the *Rayleigh resolution*. See J. D. Kraus, *Radio Astronomy,* 2d ed., pp. 6–19, Cygnus-Quasar, 1986.

satellites in the Clarke geostationary orbit separated by 1°. Thus, when the antenna beam maximum is aligned with one satellite, the first null coincides with the adjacent satellite.

Half the beamwidth between first nulls is approximately equal to the half-power beamwidth (HPBW) or

$$\frac{\text{FNBW}}{2} \cong \text{HPBW} \tag{1}$$

Thus, from (2–4–6) the product of the FNBW/2 in the two principal planes of the antenna pattern is a measure of the antenna beam area.[1] Thus,

$$\Omega_A = \left(\frac{\text{FNBW}}{2}\right)_\theta \left(\frac{\text{FNBW}}{2}\right)_\phi \tag{2}$$

It then follows that the number $N$ of radio transmitters or point sources of radiation distributed uniformly over the sky which an antenna can resolve is given approximately by

$$N = \frac{4\pi}{\Omega_A} \tag{3}$$

where $\Omega_A =$ beam area, sr

However, from (2–7–4),

$$D = \frac{4\pi}{\Omega_A} \tag{4}$$

and we may conclude that *ideally* the number of point sources an antenna can resolve is numerically equal to the directivity of the antenna or

$$\boxed{D = N} \tag{5}$$

Equation (4) states that the directivity is equal to the number of beam areas into which the antenna pattern can subdivide the sky and (5) gives the added significance that the ***directivity is equal to the number of point sources in the sky that the antenna can resolve*** under the assumed ideal conditions of a uniform source distribution.[2]

## 2–9 ANTENNA APERTURES

The concept of aperture is most simply introduced by considering a receiving antenna. Suppose that the receiving antenna is a rectangular electromagnetic horn immersed in the field of a uniform plane wave as suggested in Fig. 2–7. Let the Poynting vector, or power density, of the plane wave be $S$ watts per square meter and the area, or physical aperture of the horn, be $A_p$ square meters. If the horn extracts all the power from the wave over its entire physical aperture, then the total power $P$ absorbed from the wave is

$$P = \frac{E^2}{Z} A_p = S A_p \qquad \text{(W)} \tag{1}$$

[1] Usually FNBW/2 is slightly greater than the HPBW and (2) is actually a better approximation to $\Omega_A$ than $\Omega_A = \theta_{HP}\phi_{HP}$ as given by (2–4–6).

[2] A strictly regular distribution of points on a sphere is only possible for 4, 6, 8, 12, and 20 points corresponding to the vertices of a tetrahedron, cube, octahedron, icosahedron and dodecahedron.

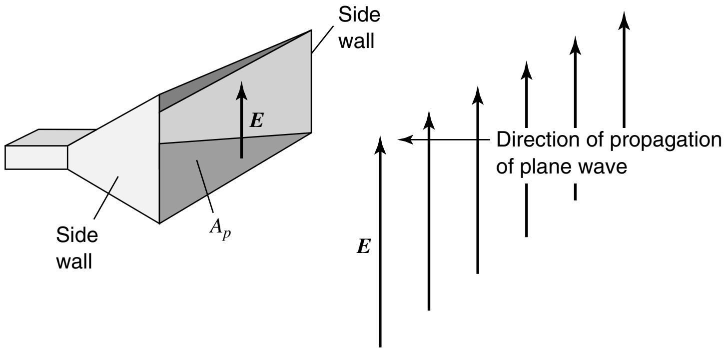



**Figure 2–7**
Plane wave incident on electromagnetic horn of physical aperture $A_p$.

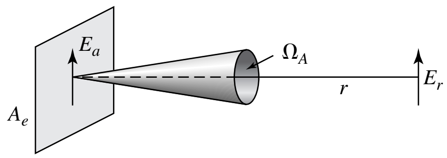



**Figure 2–8**
Radiation over beam area $\Omega_A$ from aperture $A_e$.

Thus, the electromagnetic horn may be regarded as having an aperture, the total power it extracts from a passing wave being proportional to the aperture or area of its mouth.

*But* the field response of the horn is NOT uniform across the aperture $A$ because $E$ at the sidewalls must equal zero. Thus, the *effective aperture* $A_e$ of the horn is less than the *physical aperture* $A_p$ as given by

$$\varepsilon_{ap} = \frac{A_e}{A_p} \qquad \text{(dimensionless)} \qquad \textbf{\textit{Aperture efficiency}} \tag{2}$$

where $\varepsilon_{ap}$ = aperture efficiency.

For horn and parabolic reflector antenna, aperture efficiencies are commonly in the range of 50 to 80% ($0.5 \le \varepsilon_{ap} \le 0.8$). Large dipole or patch arrays with uniform field to the edges of the physical aperture may attain higher aperture efficiencies approaching 100%. However, to reduce sidelobes, fields are commonly tapered toward the edges, resulting in reduced aperture efficiency.

Consider now an antenna with an *effective aperture* $A_e$, which radiates all of its power in a conical pattern of beam area $\Omega_A$, as suggested in Fig. 2–8. Assuming a uniform field $E_a$ over the aperture, the power radiated is

$$P = \frac{E_a^2}{Z_0} A_e \qquad \text{(W)} \tag{3}$$

where $Z_0$ = intrinsic impedance of medium (377 Ω for air or vacuum).

Assuming a uniform field $E_r$ in the far field at a distance $r$, the power radiated is also given by

$$P = \frac{E_r^2}{Z_0} r^2 \Omega_A \qquad \text{(W)} \tag{4}$$

Equating (3) and (4) and noting that $E_r = E_a A_e / r\lambda$ yields the aperture–beam-area relation

$$\boxed{\lambda^2 = A_e \Omega_A \qquad (\text{m}^2) \qquad \textbf{\textit{Aperture–beam-area relation}}} \tag{5}$$

where $\Omega_A$ = beam area (sr).

Thus, if $A_e$ is known, we can determine $\Omega_A$ (or vice versa) at a given wavelength. From (5) and (2–7–4) it follows that the directivity

$$\boxed{D = 4\pi \frac{A_e}{\lambda^2} \qquad \textbf{\textit{Directivity from aperture}}} \tag{6}$$

All antennas have an effective aperture which can be calculated or measured. Even the hypothetical, idealized isotropic antenna, for which $D = 1$, has an effective aperture

$$A_e = \frac{D\lambda^2}{4\pi} = \frac{\lambda^2}{4\pi} = 0.0796\lambda^2 \tag{7}$$

All lossless antennas must have an effective aperture equal to or greater than this. By reciprocity the effective aperture of an antenna is the same for receiving and transmitting.

Three expressions have now been given for the *directivity* $D$. They are

$$\boxed{D = \frac{P(\theta, \phi)_{\text{max}}}{P(\theta, \phi)_{\text{av}}} \qquad (\text{dimensionless}) \qquad \textbf{\textit{Directivity from pattern}}} \tag{8}$$

$$\boxed{D = \frac{4\pi}{\Omega_A} \qquad (\text{dimensionless}) \qquad \textbf{\textit{Directivity from pattern}}} \tag{9}$$

$$\boxed{D = 4\pi \frac{A_e}{\lambda^2} \qquad (\text{dimensionless}) \qquad \textbf{\textit{Directivity from aperture}}} \tag{10}$$

When the antenna is receiving with a load resistance $R_L$ matched to the antenna radiation resistance $R_r$ ($R_L = R_r$), as much power is reradiated from the antenna as is delivered to the load. This is the condition of *maximum power transfer* (antenna assumed lossless).

In the circuit case of a load matched to a generator, as much power is dissipated in the generator as is delivered to the load. Thus, for the case of the dipole antenna in Fig. 2–9 we have a *load power*

$$P_{\text{load}} = SA_e \qquad (\text{W}) \tag{11}$$

where

$S$ = power density at receiving antenna, W/m$^2$
$A_e$ = effective aperture of antenna, m$^2$

and a *reradiated power*

$$P_{\text{rerad}} = \frac{\text{Power reradiated}}{4\pi \text{ sr}} = SA_r \qquad (\text{W})$$

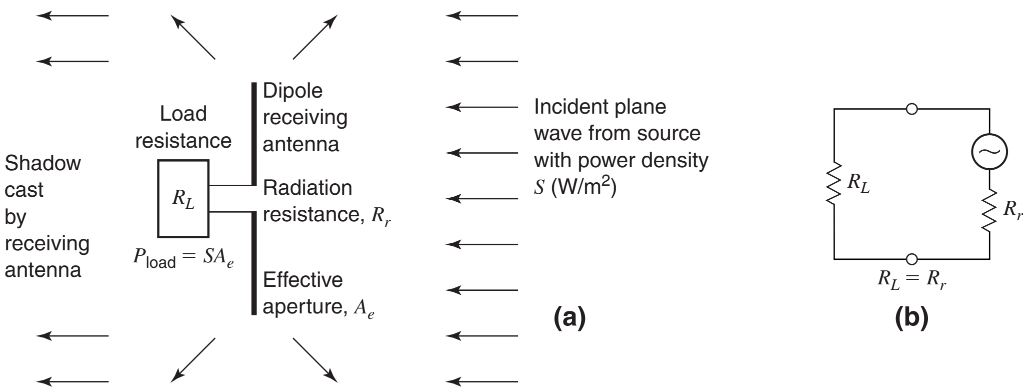



**Figure 2–9**
(*a*) The receiving antenna matched to a load ($R_r = R_L$) reradiates a power that is equal to the power delivered to the load. More generally, the reradiated and scattered power from any antenna or object yields a *radar cross-section* (RCS) which is proportional to the back-scattered power received at a radar at a distance $r$, as discussed in Chapter 12. (*b*) Equivalent circuit.

where $A_r$ = reradiating aperture $= A_e$, m$^2$ and

$$P_{\text{rerad}} = P_{\text{load}}$$

The above discussion is applicable to a single dipole ($\lambda/2$ or shorter). However, it does not apply to all antennas. In addition to the reradiated power, an antenna may scatter power that does not enter the antenna-load circuit. Thus, the reradiated plus scattered power may exceed the power delivered to the load. See Sec. 21–15 for a discussion that includes both receiving and transmitting conditions.

## 2–10 EFFECTIVE HEIGHT

The *effective height h* (meters) of an antenna is another parameter related to the aperture. Multiplying the effective height by the incident field $E$ (volts per meter) of the same polarization gives the voltage $V$ induced. Thus,

$$V = hE \tag{1}$$

Accordingly, the effective height may be defined as the ratio of the induced voltage to the incident field or

$$h = \frac{V}{E} \qquad \text{(m)} \tag{2}$$

Consider, for example, a vertical dipole of length $l = \lambda/2$ immersed in an incident field $E$, as in Fig. 2–9–1($a$). If the current distribution of the dipole were uniform, its effective height would be $l$. The actual current distribution, however, is nearly sinusoidal with an *average value* $2/\pi = 0.64$ (of the maximum) so that its effective height $h = 0.64\,l$. It is assumed that the antenna is oriented for maximum response.

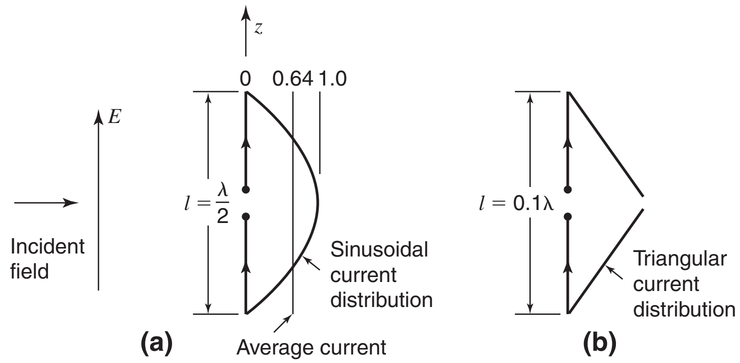



**Figure 2–9–1**
(*a*) Dipole of length $l = \lambda/2$ with sinusoidal current distribution.
(*b*) Dipole of length $l = 0.1\lambda$ with triangular current distribution.

If the same dipole is used at a longer wavelength so that it is only $0.1\lambda$ long, the current tapers almost linearly from the central feed point to zero at the ends in a triangular distribution, as in Fig. 2–9–1(*b*). The average current is $1/2$ of the maximum so that the effective height is $0.5l$.

Thus, another way of defining effective height is to consider the transmitting case and equate the effective height to the physical height (or length $l$) multiplied by the (normalized) average current or

$$h_e = \frac{1}{I_0}\int_0^{h_p} I(z)\,dz = \frac{I_{\text{av}}}{I_0}h_p \qquad \text{(m)} \tag{3}$$

where

$h_e$ = effective height, m
$h_p$ = physical height, m
$I_{\text{av}}$ = average current, A

It is apparent that *effective height* is a useful parameter for transmitting tower-type antennas.[1] It also has an application for small antennas. The parameter *effective aperture* has more general application to all types of antennas. The two have a simple relation, as will be shown.

For an antenna of radiation resistance $R_r$ matched to its load, the power delivered to the load is equal to

$$P = \frac{1}{4}\frac{V^2}{R_r} = \frac{h^2E^2}{4R_r} \qquad \text{(W)} \tag{4}$$

[1]Effective height can also be expressed more generally as a vector quantity. Thus (for linear polarization) we can write

$$V = \mathbf{h}_e \cdot \mathbf{E} = h_e E \cos\theta$$

where

$\mathbf{h}_e$ = effective height and polarization angle of antenna, m
$\mathbf{E}$ = field intensity and polarization angle of incident wave, V m$^{-1}$
$\theta$ = angle between polarization angles of antenna and wave, deg

In a still more general expression (for any polarization state), $\theta$ is the angle between polarization states on the Poincaré sphere (see Sec. 2–17).

In terms of the effective aperture the same power is given by

$$P = SA_e = \frac{E^2 A_e}{Z_0} \qquad \text{(W)} \tag{5}$$

where $Z_0$ = intrinsic impedance of space (= 377 Ω)

Equating (4) and (5), we obtain

$$h_e = 2\sqrt{\frac{R_r A_e}{Z_0}} \qquad \text{(m)} \qquad \text{and} \qquad A_e = \frac{h_e^2 Z_0}{4R_r} \qquad (\text{m}^2) \tag{6}$$

Thus, effective height and effective aperture are related via radiation resistance and the intrinsic impedance of space.

To summarize, we have discussed the space parameters of an antenna, namely, field and power patterns, beam area, directivity, gain, and various apertures. We have also discussed the circuit quantity of radiation resistance and alluded to antenna temperature, which is discussed further in Sec. 12–1. Figure 2–10 illustrates this ***duality*** of an antenna.

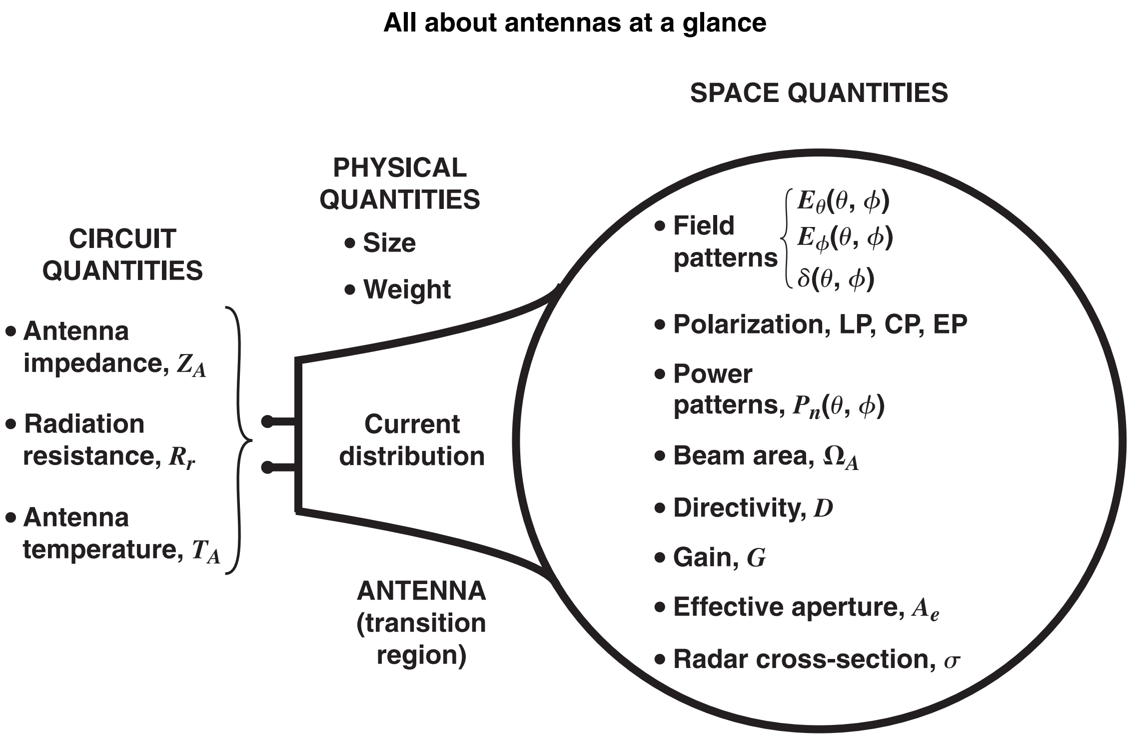



**Figure 2–10**
The parameters or terminology of antennas illustrating their *duality* as a *circuit device* (with resistance and temperature) on one hand and a *space device* (with patterns, polarization, beam area, directivity, gain, aperture and radar cross-section) on the other. Other antenna quantities are its physical size and bandwidth (involving impedance, *Q* and pattern).

EXAMPLE

## 2–10.1 Effective Aperture and Directivity of a Short Dipole Antenna

A plane wave is incident on a short dipole as in Fig. 2–11. The wave is assumed to be linearly polarized with $E$ in the $y$ direction. The current on the dipole is assumed constant and in the same phase over its entire length, and the terminating resistance $R_T$ is assumed equal to the dipole radiation resistance $R_r$. The antenna loss resistance $R_L$ is assumed equal to zero. What is ($a$) the dipole's maximum effective aperture and ($b$) its directivity?

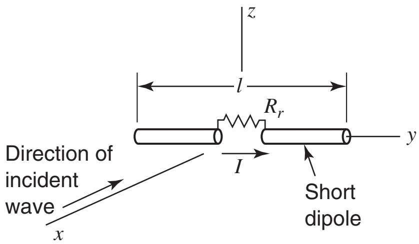



**Figure 2–11**
Short dipole with uniform current induced by incident wave.

**Solution**

($a$) The maximum effective aperture of an antenna is

$$A_{em} = \frac{V^2}{4SR_r} \tag{7}$$

where the effective value of the induced voltage $V$ is here given by the product of the effective electric field intensity at the dipole and its length, that is,

$$V = El \tag{8}$$

The radiation resistance $R_r$ of a short dipole of length $l$ with uniform current will be shown later to be

$$R_r = \frac{80\pi^2 l^2}{\lambda^2}\left(\frac{I_{av}}{I_0}\right)^2 = 790\left(\frac{I_{av}}{I_0}\right)^2\left(\frac{l}{\lambda}\right)^2 \quad (\Omega) \tag{9}$$

where

$\lambda$ = wavelength
$I_{av}$ = average current
$I_0$ = terminal current

The power density, or Poynting vector, of the incident wave at the dipole is related to the field intensity by

$$S = \frac{E^2}{Z} \tag{10}$$

where $Z$ = intrinsic impedance of the medium.

In the present case, the medium is free space so that $Z = 120\pi\ \Omega$. Now substituting (8), (9), and (10) into (7), we obtain for the maximum effective aperture of a short dipole (for $I_{av} = I_0$)

$$A_{em} = \frac{120\pi E^2 l^2 \lambda^2}{320\pi^2 E^2 l^2} = \frac{3}{8\pi}\lambda^2 = 0.119\lambda^2 \qquad \textit{Ans. (a)}$$

(*b*)

$$D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \times 0.119\lambda^2}{\lambda^2} = 1.5 \qquad \textit{Ans. (b)}$$

A typical short dipole might be $\lambda/10$ long and $\lambda/100$ in diameter for a physical cross-sectional aperture of $0.001\lambda^2$ as compared to the $0.119\lambda^2$ effective aperture of Example 2–10.1. Thus, a single dipole or linear antenna may have a physical aperture that is smaller than its effective aperture. However, a *broadside array* of many dipoles or linear antennas has an *overall* physical aperture that, like horns and dishes, is larger than its effective aperture. On the other hand, an end-fire array of dipoles, as in a Yagi-Uda antenna, has an *end-on* physical cross section that is smaller than the antenna's effective aperture. Thus, depending on the antenna, physical apertures may be larger than effective apertures, or vice versa.

**EXAMPLE**

### 2–10.2 Effective Aperture and Directivity of Linear $\lambda/2$ Dipole

A plane wave incident on the antenna is traveling in the negative $x$ direction as in Fig. 2–12*a*. The wave is linearly polarized with $E$ in the $y$ direction. The equivalent circuit is shown in Fig. 2–12*b*. The antenna has been replaced by an equivalent or Thévenin generator. The infinitesimal voltage $dV$ of this generator due to the voltage induced by the incident wave in an infinitesimal element of length $dy$ of the antenna is

$$dV = E\,dy \cos\frac{2\pi y}{\lambda} \tag{11}$$

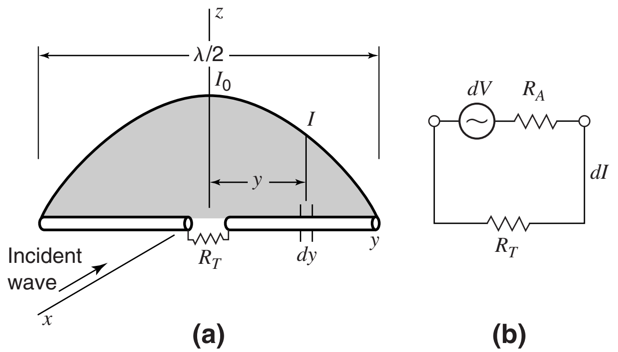



**Figure 2–12**
Linear $\lambda/2$ antenna in field of electromagnetic wave (*a*) and equivalent circuit (*b*).

It is assumed that the infinitesimal induced voltage is proportional to the current at the infinitesimal element as given by the current distribution (11). Find (*a*) the effective aperture and (*b*) the directivity of the $\lambda/2$ dipole.

**Solution**

(*a*) The total induced voltage $V$ is given by integrating (11) over the length of the antenna. This may be written as

$$V = 2\int_0^{\lambda/4} E\cos\frac{2\pi y}{\lambda}\,dy \qquad (12)$$

Performing the integration in (12) we have

$$V = \frac{E\lambda}{\pi} \qquad (13)$$

The value of the radiation resistance $R_r$ of the linear $\lambda/2$ antenna will be taken as 73 Ω. The terminating resistance $R_T$ is assumed equal to $R_r$. Thus, we obtain, for the maximum effective aperture of a linear $\lambda/2$ antenna,

$$A_{em} = \frac{120\pi E^2\lambda^2}{4\pi^2 E^2 \times 73} = \frac{30}{73\pi}\lambda^2 = 0.13\lambda^2 \qquad \textit{Ans. (a)}$$

$$(b)\quad D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \times 0.13\lambda^2}{\lambda^2} = 1.63 \qquad \textit{Ans. (b)}$$

The maximum effective aperture of the linear $\lambda/2$ antenna is about 10 percent greater than that of the short dipole.

The maximum effective aperture of the $\lambda/2$ antenna is approximately the same as an area 1/2 by $1/4\lambda$ on a side, as illustrated in Fig. 2–13*a*. This area is $0.125\lambda^2$. An elliptically shaped aperture of $0.13\lambda^2$ is shown in Fig. 2–13*b*. The physical significance of these apertures is that power from the incident plane wave is absorbed over an area of this size and is delivered to the terminal resistance or load.

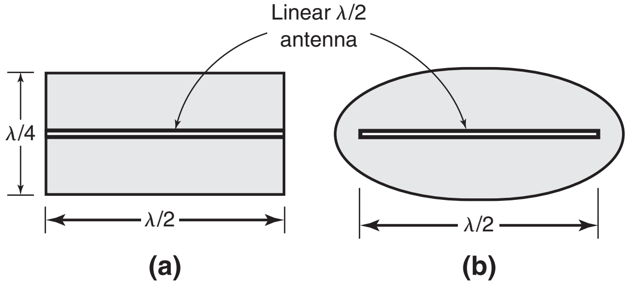



**Figure 2–13**
(*a*) Maximum effective aperture of linear $\lambda/2$ antenna is approximately represented by rectangle 1/2 by $1/4\lambda$ on a side.
(*b*) Maximum effective aperture of linear $\lambda/2$ antenna represented by elliptical area of $0.13\lambda^2$.

Although the radiation resistance, effective aperture, and directivity are the same for both receiving and transmitting, the current distribution is, in general, not the same. Thus, a plane wave incident on a receiving antenna excites a different current distribution than a localized voltage applied to a pair of terminals for transmitting.

## 2–11 THE RADIO COMMUNICATION LINK

The usefulness of the aperture concept is well illustrated by using it to derive the important Friis transmission formula published in 1946 by Harald T. Friis (1) of the Bell Telephone Laboratory.

Referring to Fig. 2–14, the formula gives the power received over a radio communication link. Assuming lossless, matched antennas, let the transmitter feed a power $P_t$ to a transmitting antenna of effective aperture $A_{et}$. At a distance $r$ a receiving antenna of effective aperture $A_{er}$ intercepts some of the power radiated by the transmitting antenna and delivers it to the receiver $R$. Assuming for the moment that the transmitting antenna is isotropic, the power per unit area available at the receiving antenna is

$$S_r = \frac{P_t}{4\pi r^2} \qquad \text{(W)} \tag{1}$$

If the antenna has gain $G_t$, the power per unit area available at the receiving antenna will be increased in proportion as given by

$$S_r = \frac{P_t G_t}{4\pi r^2} \qquad \text{(W)} \tag{2}$$

Now the power collected by the lossless, matched receiving antenna of effective aperture $A_{er}$ is

$$P_r = S_r A_{er} = \frac{P_t G_t A_{er}}{4\pi r^2} \qquad \text{(W)} \tag{3}$$

The gain of the transmitting antenna can be expressed as

$$G_t = \frac{4\pi A_{et}}{\lambda^2} \tag{4}$$

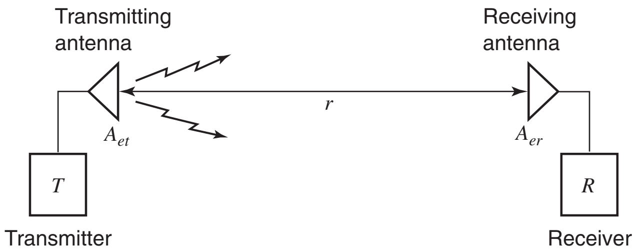



**Figure 2–14** Communication circuit with waves from transmitting antenna arriving at the receiving antenna by a direct path of length $r$.

Substituting this in (3) yields the *Friis transmission formula*

$$\frac{P_r}{P_t} = \frac{A_{er} A_{et}}{r^2 \lambda^2} \qquad \text{(dimensionless)} \qquad \textbf{\textit{Friis transmission formula}} \tag{5}$$

where

$P_r$ = received power, W
$P_t$ = transmitted power, W
$A_{et}$ = effective aperture of transmitting antenna, $m^2$
$A_{er}$ = effective aperture of receiving antenna, $m^2$
$r$ = distance between antennas, m
$\lambda$ = wavelength, m

**EXAMPLE** **2–11.1 Radio Communication Link**

A radio link has a 15-W transmitter connected to an antenna of 2.5 $m^2$ effective aperture at 5 GHz. The receiving antenna has an effective aperture of 0.5 $m^2$ and is located at a 15-km line-of-sight distance from the transmitting antenna. Assuming lossless, matched antennas, find the power delivered to the receiver.

**Solution**

From (5)

$$P = P_t \frac{A_{et} A_{er}}{r^2 \lambda^2} = 15 \frac{2.5 \times 0.5}{15^2 \times 10^6 \times 0.06^2} = 23\ \mu\text{W} \qquad \textit{Ans.}$$

## 2–12 FIELDS FROM OSCILLATING DIPOLE

Although a charge moving with uniform velocity along a straight conductor does not radiate, a charge moving back and forth in simple harmonic motion along the conductor is subject to acceleration (and deceleration) and radiates.

To illustrate radiation from a dipole antenna, let us consider that the dipole of Fig. 2–15 has two equal charges of opposite sign oscillating up and down in harmonic motion with instantaneous separation $l$ (maximum separation $l_0$) while focusing attention on the electric field. For clarity only a single electric field line is shown.

At time $t = 0$ the charges are at maximum separation and undergo maximum acceleration $\dot{v}$ as they reverse direction (Fig. 2–15*a*). At this instant the current $I$ is zero. At an $\frac{1}{8}$-period later, the charges are moving toward each other (Fig. 2–15*b*) and at a $\frac{1}{4}$-period they pass at the midpoint (Fig. 2–15*c*). As this happens, the field lines detach and new ones of opposite sign are formed. At this time the equivalent current $I$ is a maximum and the charge acceleration is zero. As time progresses to a $\frac{1}{2}$-period, the fields continue to move out as in Fig. 2–15*d* and *e*.

An oscillating dipole with more field lines is shown in Fig. 2–16 at four instants of time.

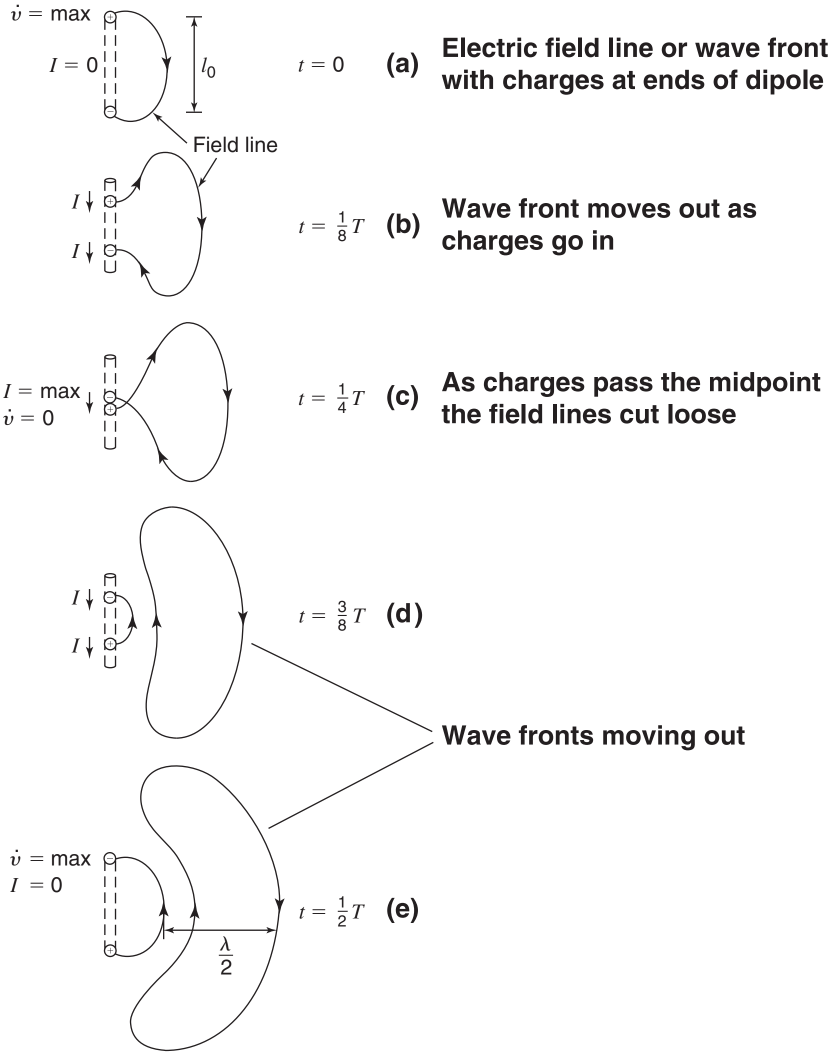



**Figure 2–15**
Oscillating electric dipole consisting of two electric charges in simple harmonic motion, showing propagation of an electric field line and its detachment (radiation) from the dipole. Arrows next to the dipole indicate current ($I$) direction.

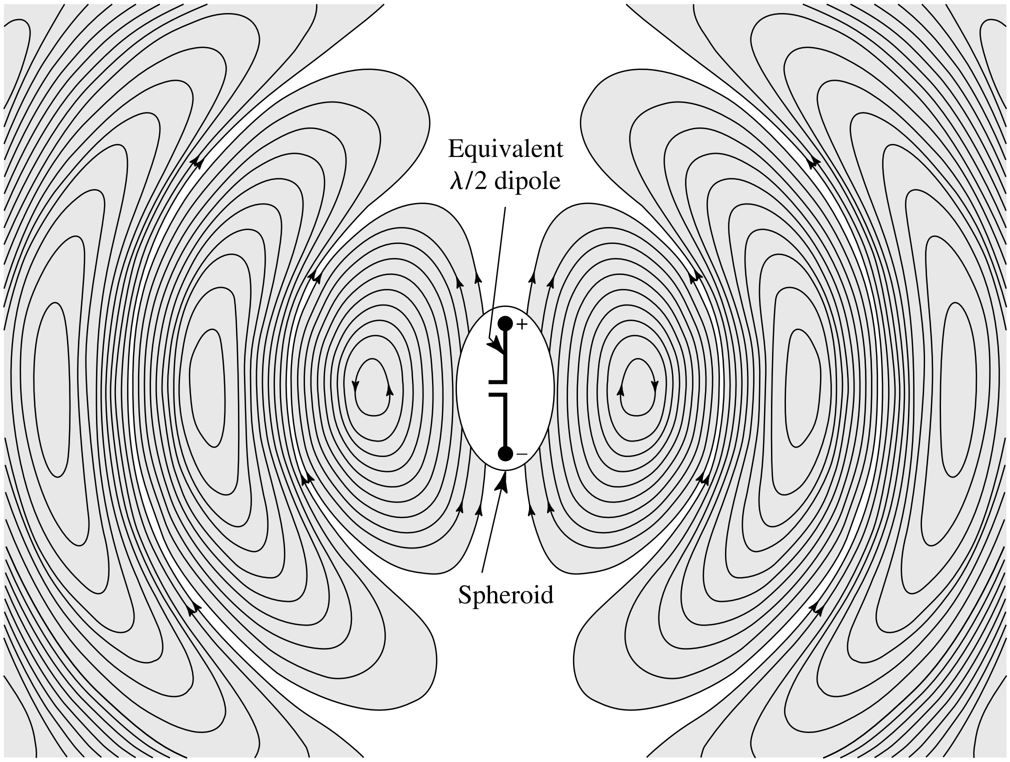



**Figure 2–16**
Electric field lines of the radiation moving out from $\lambda/2$ dipole antenna. (*Produced by Edward M. Kennaugh, courtesy of John D. Cowan, Jr.*)

## 2–13 ANTENNA FIELD ZONES

The fields around an antenna may be divided into two principal regions, one near the antenna called the *near field* or *Fresnel zone* and one at a large distance called the *far field* or *Fraunhofer zone*. Referring to Fig. 2–17, the boundary between the two may be arbitrarily taken to be at a radius

$$R = \frac{2L^2}{\lambda} \qquad \text{(m)} \tag{1}$$

where

$L$ = maximum dimension of the antenna, m
$\lambda$ = wavelength, m

In the far or Fraunhofer region, the measurable field components are transverse to the radial direction from the antenna and all power flow is directed radially outward. In the far field the shape of the field pattern is independent of the distance. In the near or Fresnel region, the longitudinal component of the electric field may be significant and power flow is not entirely radial. In the near field, the shape of the field pattern depends, in general, on the distance.

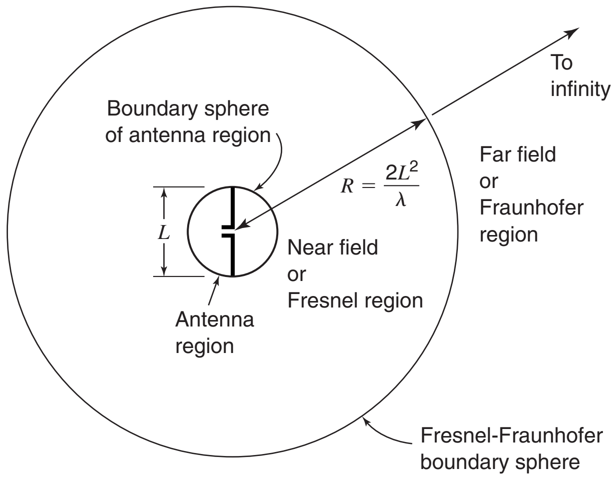



**Figure 2–17**
Antenna region, Fresnel region and Fraunhofer region.

Enclosing the antenna in an imaginary boundary sphere as in Fig. 2–18*a* it is as though the region near the poles of the sphere acts as a reflector. On the other hand, the waves expanding perpendicular to the dipole in the equatorial region of the sphere result in power leakage through the sphere as if partially transparent in this region.

This results in reciprocating (oscillating) energy flow near the antenna accompanied by outward flow in the equatorial region. The outflow accounts for the power radiated from the antenna, while the reciprocating energy represents reactive power that is trapped near the antenna like in a resonator. This oversimplified discussion accounts in a qualitative way for the field pattern of the $\lambda/2$ dipole antenna as shown in Fig. 2–18*b*. The energy picture is discussed in more detail in Sec. 6–2 and displayed in Fig. 6–6.

For a $\lambda/2$ dipole antenna, the energy is stored at one instant of time in the electric field, mainly near the ends of the antenna or maximum charge regions, while a $\frac{1}{2}$-period later the energy is stored in the magnetic field mainly near the center of the antenna or maximum current region.

Note that although the term *power flow* is sometimes used, it is actually *energy* which flows, power being the time rate of energy flow. A similar loose usage occurs when we say we pay a power bill, when, in fact, we are actually paying for electric energy.

## 2–14 SHAPE-IMPEDANCE CONSIDERATIONS

It is possible in many cases to deduce the qualitative behavior of an antenna from its shape. This may be illustrated with the aid of Fig. 2–19. Starting with the opened-out two-conductor transmission line of Fig. 2–19*a*, we find that, if extended far enough, a nearly constant impedance will be provided at the input (left) end for $d \ll \lambda$ and $D \geq \lambda$.

In Fig. 2–19*b* the curved conductors are straightened into regular cones and in Fig. 2–19*c* the cones are aligned collinearly, forming a biconical antenna. In Fig. 2–19*d* the cones degenerate into straight wires. In going from Fig. 2–19*a* to *d*, the bandwidth of relatively constant impedance tends to decrease. Another difference is that the antennas of Fig. 2–19*a*

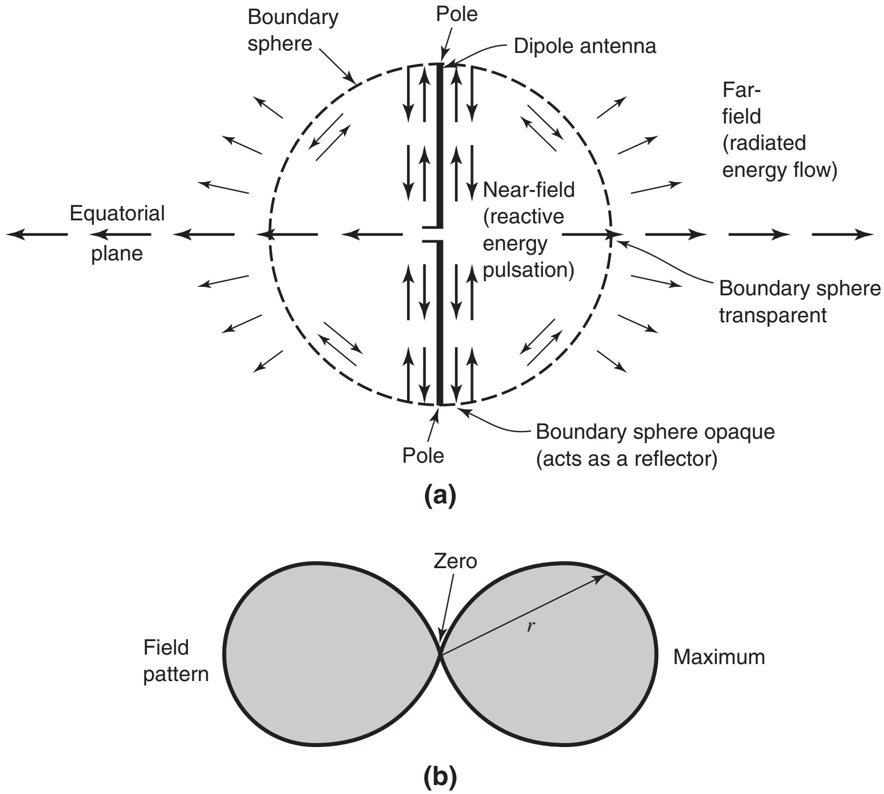



**Figure 2–18**
Energy flow near a dipole antenna (*a*) and radiation field pattern (*b*). The radius vector *r* is proportional to the field radiated in that direction.

and *b* are unidirectional with beams to the right, while the antennas of Fig. 2–19*c* and *d* are omnidirectional in the horizontal plane (perpendicular to the wire or cone axes).

A different modification is shown in Fig. 2–19*e*. Here the two conductors are curved more sharply and in opposite directions, resulting in a spiral antenna with maximum radiation broadside (perpendicular to the page) and with polarization which rotates clockwise. This antenna, like the one in Fig. 2–19*a*, exhibits very broadband characteristics (see Chap. 11).

The dipole antennas of Fig. 2–19 are balanced, i.e., they are fed by two-conductor (balanced) transmission lines. Figure 2–20 illustrates a similar evolution of monopole antennas, i.e., antennas fed from coaxial (unbalanced) transmission lines.

By gradually tapering the inner and outer conductors or a coaxial transmission line, a very wide band antenna with an appearance reminiscent of a volcanic crater and puff of smoke is obtained, as suggested in the cutaway view of Fig. 2–20*a*.

In Fig. 2–20*b* the volcano form is modified into a double dish and in Fig. 2–20*c* into two wide-angle cones. All of these antennas are omnidirectional in a plane perpendicular

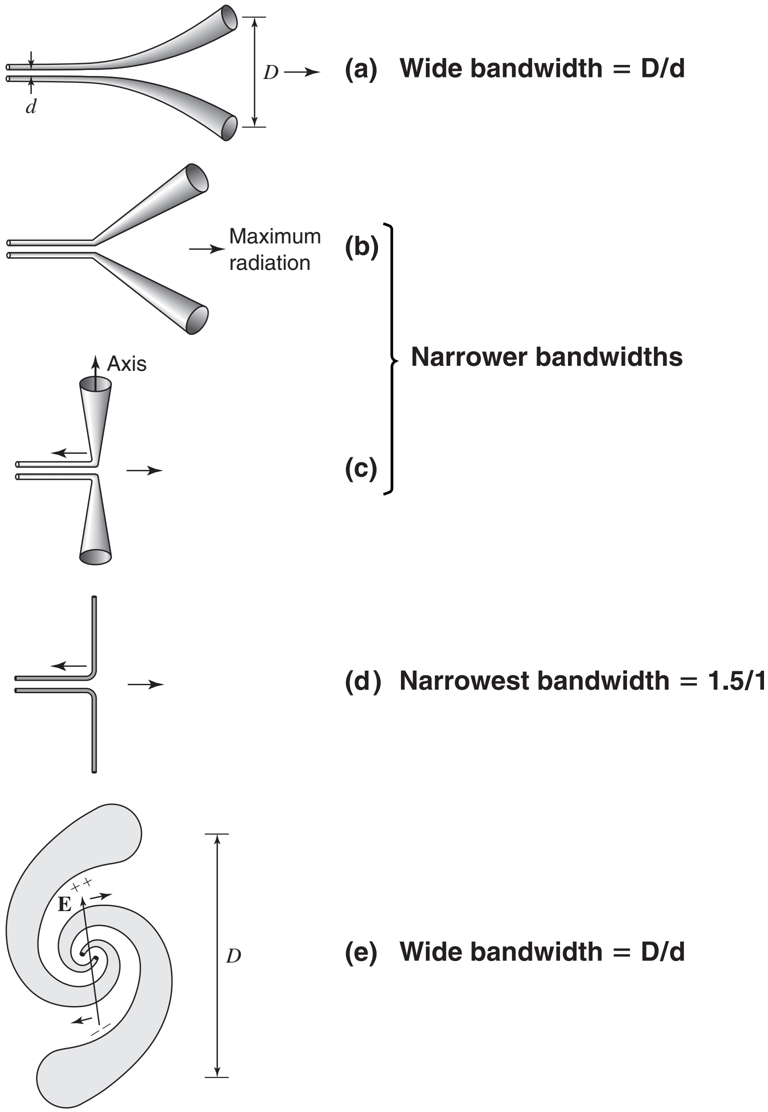



**Figure 2–19** Evolution of a thin cylindrical antenna (*d*) from an opened-out twin line (*a*). Curving the conductors as in (*e*) results in the spiral antenna.

to their axes and all have a wide bandwidth. For example, an actual biconical antenna, as in Fig. 2–20*c*, with a full cone angle of 120° has an omnidirectional pattern and nearly constant 50-Ω input impedance (power reflection less than 1 percent or VSWR $< 1.2$) over a 6 to 1 bandwidth with cone diameter $D = \lambda$ at the lowest frequency.

Increasing the lower cone angle to 180° or into a flat ground plane while reducing the upper cone angle results in the antenna of Fig. 2–20*d*. Collapsing the upper cone into a thin stub, we arrive at the extreme modification of Fig. 2–20*e*. If the antenna of Fig. 2–20*a* is regarded as the most basic form, the stub type of Fig. 2–20*e* is the most degenerate form, with a relatively narrow bandwidth.

| | |
|---|---|
| Antennas with large and abrupt discontinuities have large reflections and act as reflectionless transducers only over narrow frequency bands where the reflections cancel. | Antennas with discontinuities that are small and gradual have small reflections and are, in general, relatively reflectionless transducers over wide frequency bands. |

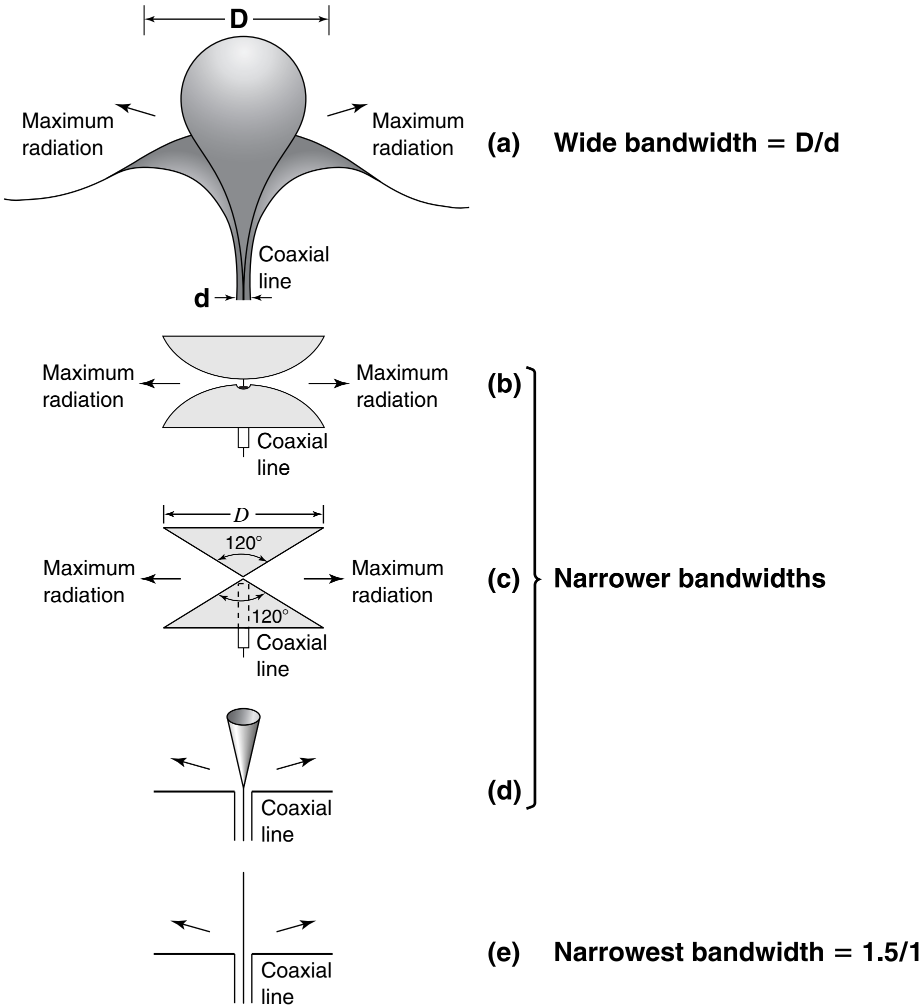



**Figure 2–20**
Evolution of stub (monopole) antenna (*e*) from volcano-smoke antenna (*a*). Fed from coaxial (unbalanced) transmission lines.

As we depart further from the basic type, the discontinuity in the transmission line becomes more abrupt at what eventually becomes the junction of the ground plane and the coaxial line. This discontinuity results in some energy being reflected back into the line. The reflection at the end of the antenna also increases for thinner antennas. At some frequency the two reflections may compensate, but the bandwidth of compensation is narrow.

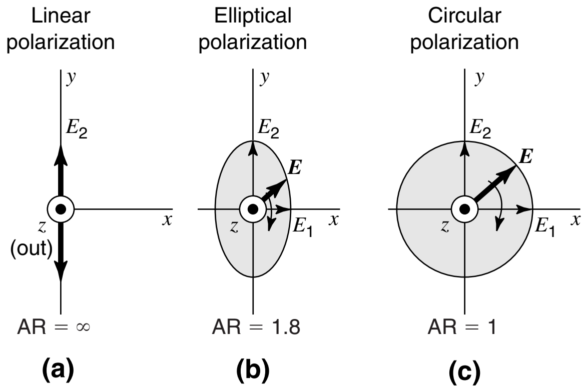



**Figure 2–21**
(*a*) Linear, (*b*) elliptical, and (*c*) circular polarization for left-circularly polarized wave approaching.

## 2–15 LINEAR, ELLIPTICAL AND CIRCULAR POLARIZATION[1]

Consider a plane wave traveling out of the page (positive $z$ direction), as in Fig. 2–21*a*, with the electric field at all times in the $y$ direction. This wave is said to be *linearly polarized* (in the $y$ direction). As a function of time and position, the electric field is given by

$$E_y = E_2 \sin(\omega t - \beta z) \tag{1}$$

In general, the electric field of a wave traveling in the $z$ direction may have both a $y$ component and an $x$ component, as suggested in Fig. 2–21*b*. In this more general situation, with a phase difference $\delta$ between the components, the wave is said to be *elliptically polarized*. At a fixed value of $z$ the electric vector **E** rotates as a function of time, the tip of the vector describing an ellipse called the ***polarization ellipse***. The ratio of the major to minor axes of the polarization ellipse is called the ***Axial Ratio*** (AR). Thus, for the wave Fig. 2–21*b*, AR $= E_2/E_1$. Two extreme cases of elliptical polarization correspond to *circular polarization,* as in Fig. 2–21*c*, and *linear polarization,* as in Fig. 2–21*a*. *For circular polarization* $E_1 = E_2$ and AR $= 1$, while *for linear polarization* $E_1 = 0$ and AR $= \infty$.

In the most general case of elliptical polarization, the polarization ellipse may have any orientation, as suggested in Fig. 2–22. The elliptically polarized wave may be expressed in terms of two linearly polarized components, one in the $x$ direction and one in the $y$ direction. Thus, if the wave is traveling in the positive $z$ direction (out of the page), the electric field components in the $x$ and $y$ directions are

$$E_x = E_1 \sin(\omega t - \beta z) \tag{2}$$

$$E_y = E_2 \sin(\omega t - \beta z + \delta) \tag{3}$$

---

[1]A much more detailed and complete discussion of wave polarization is given by J. D. Kraus, *Radio Astronomy,* 2d ed., Cygnus-Quasar, Powell, Ohio, 1986.

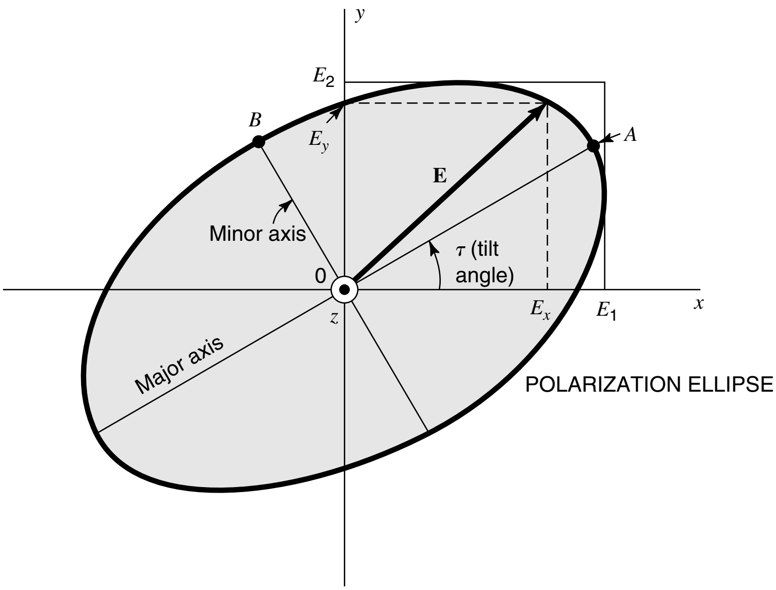



**Figure 2–22**
Polarization ellipse at tilt angle $\tau$ showing instantaneous components $E_x$ and $E_y$ and amplitudes (or peak values) $E_1$ and $E_2$.

where

$E_1$ = amplitude of wave linearly polarized in $x$ direction
$E_2$ = amplitude of wave linearly polarized in $y$ direction
$\delta$ = time-phase angle by which $E_y$ leads $E_x$

Combining (2) and (3) gives the instantaneous total vector field **E**:

$$\mathbf{E} = \hat{\mathbf{x}}E_1 \sin(\omega t - \beta z) + \hat{\mathbf{y}}E_2 \sin(\omega t - \beta z + \delta) \tag{4}$$

At $z = 0$, $E_x = E_1 \sin \omega t$ and $E_y = E_2 \sin(\omega t + \delta)$. Expanding $E_y$ yields

$$E_y = E_2(\sin \omega t \cos \delta + \cos \omega t \sin \delta) \tag{5}$$

From the relation for $E_x$ we have $\sin \omega t = E_x/E_1$ and $\cos \omega t = \sqrt{1 - (E_x/E_1)^2}$. Introducing these in (5) eliminates $\omega t$, and, on rearranging, we obtain

$$\frac{E_x^2}{E_1^2} - \frac{2E_x E_y \cos \delta}{E_1 E_2} + \frac{E_y^2}{E_2^2} = \sin^2 \delta \tag{6}$$

or

$$aE_x^2 - bE_x E_y + cE_y^2 = 1 \tag{7}$$

where

$$a = \frac{1}{E_1^2 \sin^2 \delta} \qquad b = \frac{2\cos \delta}{E_1 E_2 \sin^2 \delta} \qquad c = \frac{1}{E_2^2 \sin^2 \delta}$$

Equation (7) describes a (polarization) ellipse, as in Fig. 2–22. The line segment *OA* is the semimajor axis, and the line segment *OB* is the semiminor axis. The tilt angle of the ellipse

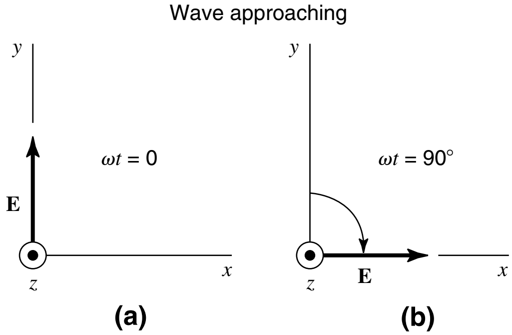



**Figure 2–23** Instantaneous orientation of electric field vector **E** at two instants of time for a left-circularly polarized wave which is approaching (out of page).

is $\tau$. The axial ratio is

$$\boxed{\text{AR} = \frac{OA}{OB} \qquad (1 \le \text{AR} \le \infty) \qquad \textbf{\textit{Axial Ratio}}} \tag{8}$$

If $E_1 = 0$, the wave is linearly polarized in the $y$ direction. If $E_2 = 0$, the wave is linearly polarized in the $x$ direction. If $\delta = 0$ and $E_1 = E_2$, the wave is also linearly polarized but in a plane at an angle of 45° with respect to the $x$ axis ($\tau = 45°$).

If $E_1 = E_2$ and $\delta = \pm 90°$, the wave is circularly polarized. When $\delta = +90°$, the wave is *left circularly polarized,* and when $\delta = -90°$, the wave is *right circularly polarized*. For the case $\delta = +90°$ and for $z = 0$ and $t = 0$, we have from (2) and (3) that $\mathbf{E} = \hat{\mathbf{y}} E_2$, as in Fig. 2–23*a*. One-quarter cycle later ($\omega t = 90°$), $\mathbf{E} = \hat{\mathbf{x}} E_1$, as in Fig. 4–23*b*. Thus, at a fixed position ($z = 0$) the electric field vector rotates clockwise (viewing the wave approaching). According to the IEEE definition, this corresponds to left circular polarization. The opposite rotation direction ($\delta = -90°$) corresponds to right circular polarization.

If the wave is viewed receding (from negative $z$ axis in Fig. 2–23), the electric vector appears to rotate in the opposite direction. Hence, clockwise rotation of **E** with the wave approaching is the same as counterclockwise rotation with the wave receding. Thus, unless the wave direction is specified, there is a possibility of ambiguity as to whether the wave is left- or right-handed. This can be avoided by defining the polarization with the aid of an axial-mode helical antenna. Thus, a right-handed helical antenna radiates (or receives) right circular (IEEE) polarization.[1] A right-handed helix, like a right-handed screw, is right-handed regardless of the position from which it is viewed. There is no possibility here of ambiguity.

The Institute of Electrical and Electronics Engineers (IEEE) definition is opposite to the classical optics definition which had been in use for centuries. The intent of the IEEE Standards Committee was to make the IEEE definition agree with the classical optics definition, but it got turned around so now we have two definitions. In this book we use the IEEE definition, which has the advantage of agreement with helical antennas as noted above.

[1] A left-handed helical antenna radiates (or receives) left circular (IEEE) polarization.

## 2–16 POYNTING VECTOR FOR ELLIPTICALLY AND CIRCULARLY POLARIZED WAVES

In complex notation the Poynting vector is

$$\mathbf{S} = \frac{1}{2}\mathbf{E} \times \mathbf{H}^* \tag{1}$$

The average Poynting vector is the real part of (1), or

$$\mathbf{S}_{\text{av}} = \text{Re}\,\mathbf{S} = \frac{1}{2}\text{Re}\,\mathbf{E} \times \mathbf{H}^* \tag{2}$$

We can also write

$$\boxed{\mathbf{S}_{\text{av}} = \frac{1}{2}\hat{\mathbf{z}}\frac{E_1^2 + E_2^2}{Z_0} = \frac{1}{2}\hat{\mathbf{z}}\frac{E^2}{Z_0} \qquad \textit{Average Poynting vector}} \tag{3}$$

where $E = \sqrt{E_1^2 + E_2^2}$ is the amplitude of the total **E** field.

**EXAMPLE 2–16.1 Elliptically Polarized Wave Power**

An elliptically polarized wave traveling in the positive $z$ direction in air has $x$ and $y$ components:

$$E_x = 3\sin(\omega t - \beta x) \qquad (\text{V m}^{-1})$$
$$E_y = 6\sin(\omega t - \beta x + 75^\circ) \qquad (\text{V m}^{-1})$$

Find the average power per unit area conveyed by the wave.

**Solution**

The average power per unit area is equal to the average Poynting vector, which from (3) has a magnitude

$$\mathbf{S}_{\text{av}} = \frac{1}{2}\frac{E^2}{Z} = \frac{1}{2}\frac{E_1^2 + E_2^2}{Z}$$

From the stated conditions, the amplitude $E_1 = 3\text{ V m}^{-1}$ and the amplitude $E_2 = 6\text{ V m}^{-1}$. Also for air $Z = 377\ \Omega$. Hence

$$\mathbf{S}_{\text{av}} = \frac{1}{2}\frac{3^2 + 6^2}{377} = \frac{1}{2}\frac{45}{377} \approx 60\text{ mW m}^{-2} \qquad \textit{Ans.}$$

## 2–17 THE POLARIZATION ELLIPSE AND THE POINCARÉ SPHERE

In the Poincaré sphere representation of wave polarization, the *polarization state* is described by a point on a sphere where the longitude and latitude of the point are related to parameters of the polarization ellipse (see Fig. 2–24) as follows:

$$\begin{aligned} \text{Longitude} &= 2\tau \\ \text{Latitude} &= 2\varepsilon \end{aligned} \tag{1}$$

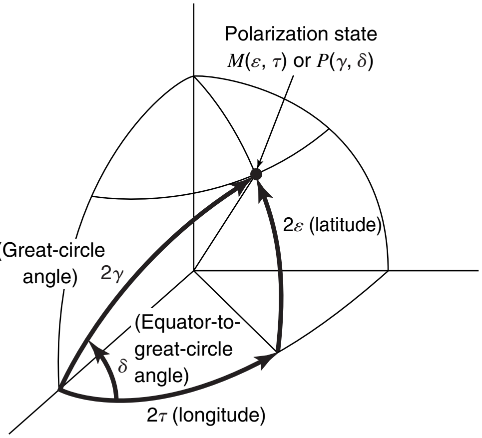



**Figure 2–24**
Poincaré sphere showing relation of angles $\varepsilon$, $\tau$, $\delta$, $\gamma$.

where $\tau$ = tilt angle, $0^\circ \le \tau \le 180^\circ$,[1] and $\varepsilon = \tan^{-1}(1/\mp\text{AR})$, $-45^\circ \le \varepsilon \le +45^\circ$. The Axial Ratio (AR) and angle $\varepsilon$ are negative for right-handed and positive for left-handed (IEEE) polarization (Poincaré-1, Deschamps-1).

The polarization state described by a point on a sphere here can also be expressed in terms of the angle subtended by the great circle drawn from a reference point on the equator and the angle between the great circle and the equator (see Fig. 2–24) as follows:

$$\begin{aligned}\text{Great-circle angle} &= 2\gamma \\ \text{Equator-to-great-circle angle} &= \delta\end{aligned} \tag{2}$$

where $\gamma = \tan^{-1}(E_2/E_1)$, $0^\circ \le \gamma \le 90^\circ$, and $\delta$ = phase difference between $E_y$ and $E_x$, $-180^\circ \le \delta \le +180^\circ$.

The geometric relation of $\tau$, $\varepsilon$, and $\gamma$ to the polarization ellipse is illustrated in Fig. 2–25. The trigonometric interrelations of $\tau$, $\varepsilon$, $\gamma$, and $\delta$ are as follows:[2]

$$\boxed{\begin{aligned}\cos 2\gamma &= \cos 2\varepsilon \cos 2\tau \\ \tan\delta &= \frac{\tan 2\varepsilon}{\sin 2\tau} \\ \tan 2\tau &= \tan 2\gamma \cos\delta \\ \sin 2\varepsilon &= \sin 2\gamma \sin\delta\end{aligned} \qquad \textbf{\textit{Polarization parameters}}} \tag{3}$$

Knowing $\varepsilon$ and $\tau$, one can determine $\gamma$ and $\delta$ or vice versa. It is convenient to describe the ***polarization state*** by either of the two sets of angles $(\varepsilon, \tau)$ or $(\gamma, \delta)$ which describe a point on the Poincaré sphere (Fig. 2–24). Let the polarization state as a function of $\varepsilon$ and $\tau$ be

---

[1]Note that when the Greek letter $\tau$ is used in this section to denote the *tilt angle,* it is associated with the incident, reflected, or transmitted wave (that is, $\tau_i$, $\tau_r$, or $\tau_t$), but when it is used to denote the *transmission coefficient,* it is associated with either a parallel or perpendicular case.

[2]These relations involve spherical trigonometry (see Born-1).

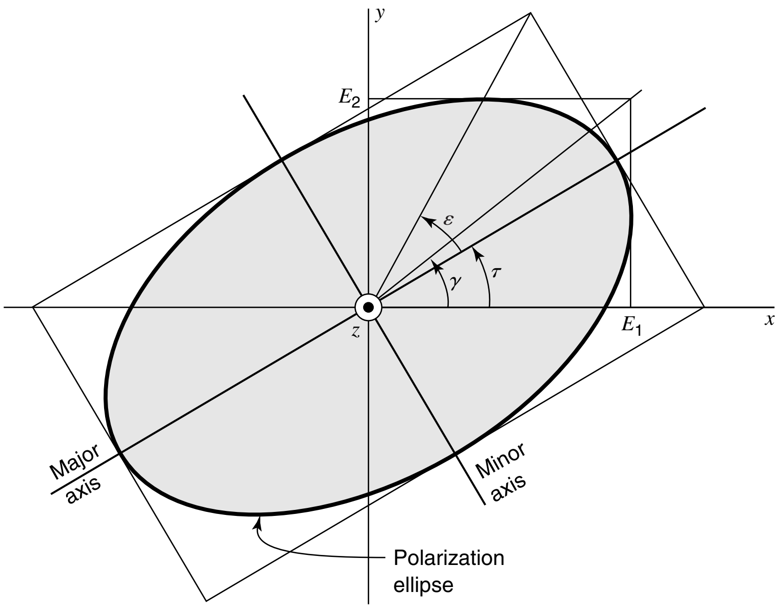



**Figure 2–25**
Polarization ellipse showing relation of angles $\varepsilon$, $\gamma$, and $\tau$.

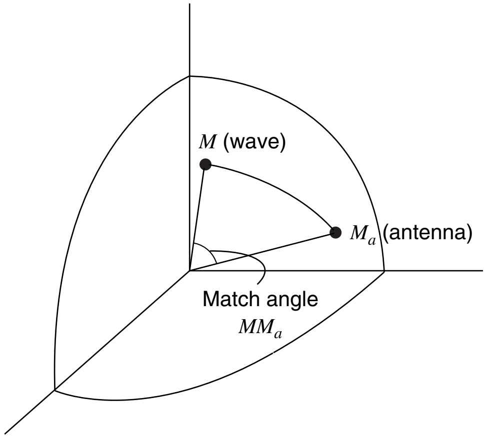



**Figure 2–26**
The match angle $MM_a$ between the polarization state of wave ($M$) and antenna ($M_a$). For $MM_a = 0°$, the match is perfect. For $MM_a = 180°$, the match is zero.

designated by $M(\varepsilon, \tau)$, or simply $M$, and the polarization state as a function of $\gamma$ and $\delta$ be designated by $P(\gamma, \delta)$, or simply $P$, as in Fig. 2–25.

As an application of the Poincaré sphere representation (see Fig. 2–26) it may be shown that the voltage response $V$ of an antenna to a wave of arbitrary polarization is given by (Sinclair-1):

$$V = k \cos \frac{MM_a}{2} \qquad \textbf{\textit{Antenna voltage response}} \tag{4}$$

where

$MM_a$ = angle subtended by great-circle line from polarization state $M$ to $M_a$
$M$ = polarization state of wave
$M_a$ = polarization state of antenna
$k$ = constant

The polarization state of the antenna is defined as the polarization state of the wave radiated by the antenna when it is transmitting. The factor $k$ in (4) involves the field strength of the wave and the size of the antenna. An important result to note is that, if $MM_a = 0°$, the antenna is matched to the wave (polarization state of wave same as for antenna) and the response is maximized. However, if $MM_a = 180°$, the response is zero. This can occur, for example, if the wave is linearly polarized in the $y$ direction while the antenna is linearly polarized in the $x$ direction; or if the wave is left circularly polarized while the antenna is right circularly polarized. More generally we may say that *an antenna is blind to a wave of opposite (or antipodal) polarization state.*

Referring to (4), a *polarization matching factor F* (for power) is given by

$$F = \cos^2 \frac{MM_a}{2} \tag{5}$$

Thus, for a perfect match the match angle $MM_a = 0°$ and $F = 1$ (states of wave and antenna the same). For a complete mismatch the match angle $MM_a = 180°$ and $F = 0$ (Fig. 2–26).

For *linear polarization,* $MM_a/2 = \Delta\tau$ and (5) reduces to

$$F = \cos^2 \Delta\tau \tag{6}$$

where $\Delta\tau$ = difference between the tilt angles of wave and antenna.

In the above discussion we have assumed a completely polarized wave, that is, one where $E_x$, $E_y$, and $\delta$ are constants. In an unpolarized wave they are not. Such a wave results when the vertical component is produced by one noise generator and the horizontal component by a different noise generator. Most cosmic radio sources are unpolarized and can be received equally well with an antenna of any polarization. If the wave is completely unpolarized, $F = \frac{1}{2}$ regardless of the state of polarization of the antenna. For a more general discussion, see J. D. Kraus, *Radio Astronomy,* 2d ed., Cygnus-Quasar, P.O. Box 85, Powell, OH 43065, 1986, Sec. 4–4.

EXAMPLE

### 2–17.1 Polarization Matching

Find the polarization matching factor $F$ for a left elliptically polarized wave ($w$) with AR = 4 and $\tau = 15°$ incident on a right elliptically polarized antenna ($a$) with AR = −2 and $\tau = 45°$.

**Solution**

From (1), $2\varepsilon(w) = 28.1°$ and $2\varepsilon(a) = -53.1°$. Thus, the wave polarization state $M$ is at latitude +28.1° and longitude 30° while the antenna polarization state $M_a$ is at latitude −53.1° and longitude 90°. Locate these positions on a globe and measure $MM_a$ with a string. The globe and string give not only the total great-circle angle $MM_a$ but also illustrate the geometry. Then compare this result with an analytical one as follows: From proportional triangles obtain $2\tau(w) = 20.7^\circ$ along the equator and $2\tau(a) = 39.3^\circ$ further along the equator. Next from (3), obtain $2\gamma(w) = 34.3^\circ$ and $2\gamma(a) = 62.4^\circ$. Thus, the total great-circle angle $MM_a = 2\gamma(w) + 2\gamma(a) = 96.7^\circ$ and the polarization matching factor

$$F = \cos^2\left(\frac{96.7}{2}\right) = 0.44$$

or the received power is 44 percent of the maximum possible value. *Ans.*

Although the radiation resistance, effective aperture, effective height, and directivity are the same for both receiving and transmitting, the current distribution is, in general, not the same. Thus, a plane wave incident on a receiving antenna excites a different current distribution than a localized voltage applied to a pair of terminals for transmitting.

### Table summarizing important relations of Chapter 2

| | |
|---|---|
| Wavelength-frequency | $\lambda = \frac{v}{f}$ (m) |
| Beam area | $\Omega_A = \iint P_n(\theta, \phi)\, d\Omega$ (sr or deg$^2$) |
| Beam area (approx.) | $\Omega_A \simeq \theta_{HP}\phi_{HP}$ (sr or deg$^2$) |
| Beam efficiency | $\varepsilon_M = \frac{\Omega_M}{\Omega_A}$ (dimensionless) |
| Directivity | $D = \frac{U(\theta,\phi)_{max}}{U_{av}} = \frac{S(\theta,\phi)_{max}}{S_{av}}$ (dimensionless) |
| Directivity | $D = \frac{4\pi}{\Omega_A}$ (dimensionless) |
| Directivity | $D = \frac{4\pi A_e}{\lambda^2}$ (dimensionless) |
| Directivity (approx.) | $D \simeq \frac{4\pi}{\theta_{HP}\phi_{HP}} \simeq \frac{41{,}000}{\theta^\circ_{HP}\phi^\circ_{HP}}$ (dimensionless) |
| Gain | $G = kD$ (dimensionless) |
| Effective aperture and beam area | $A_e\Omega_A = \lambda^2$ (m$^2$) |
| Aperture efficiency | $\varepsilon_{ap} = \frac{A_e}{A_p}$ (dimensionless) |
| Friis transmission formula | $P_r = P_t \frac{A_{et}A_{er}}{r^2\lambda^2}$ (W) |
| Current-charge continuity relation | $\dot{I}l = q\dot{v}$ (A m s$^{-1}$) |
| Radiation power | $P = \frac{\mu^2 q^2 \dot{v}^2}{6\pi Z}$ (W) |
| Near-field–far-field boundary | $R = \frac{2L^2}{\lambda}$ (m) |
| Average power per unit area of elliptically polarized wave in air | $\mathbf{S}_{av} = \frac{1}{2}\hat{\mathbf{z}}\frac{E_1^2 + E_2^2}{Z_0}$ (W m$^{-2}$) |

Table of effective aperture, directivity, effective height, and other parameters for dipoles and loops

| Antenna | Radiation resistance* $R_r$, Ω | Maximum effective aperture $A_{em}$, $\lambda^2$ | Effective height, maximum value, $h$, m | Sphere filling factor | Directivity $D$ | Directivity $D$, dBi |
|---|---|---|---|---|---|---|
| Isotropic | | $\frac{1}{4\pi} = 0.079$ | | 1 | 1 | 0 |
| Short dipole,† length $l$ | $80\left(\frac{\pi l\, I_{av}}{\lambda I_0}\right)^2$ | $\frac{3}{8\pi} = 0.119$ | $\frac{l\, I_{av}}{I_0}$ | $\frac{2}{3}$ | $\frac{3}{2}$ | 1.76 |
| Short dipole,† $l = \lambda/10$ ($I_{av} = I_0$) | 7.9 | 0.119 | $\lambda/10$ | $\frac{2}{3}$ | $\frac{3}{2}$ | 1.76 |
| Short dipole,† $l = \lambda/10$ ($I_{av} = \frac{1}{2} I_0$) | 1.98 | 0.119 | $\lambda/20$ | $\frac{2}{3}$ | $\frac{3}{2}$ | 1.76 |
| Linear, $\lambda/2$ dipole (sinusoidal current distribution) | 73 | $\frac{30}{73\pi} = 0.13$ | $\frac{\lambda}{\pi} = \frac{2l}{\pi}$ | 0.61 | 1.64 | 2.15 |
| Small loop‡ (single turn), any shape | $31{,}200\left(\frac{A}{\lambda^2}\right)^2$ | $\frac{3}{8\pi} = 0.119$ | $2\pi \frac{A}{\lambda}$ | $\frac{2}{3}$ | $\frac{3}{2}$ | 1.76 |
| Small square loop‡ (single turn), side length $= l$ Area $A = l^2 = (\lambda/10)^2$ | 3.12 | $\frac{3}{8\pi} = 0.119$ | $\frac{2\pi\lambda}{100}$ | $\frac{2}{3}$ | $\frac{3}{2}$ | 1.76 |

*See Chaps. 5 and 8.
†Length $l \le \lambda/10$.
‡Area $A \le \lambda^2/100$, see Sec. 7–9. For $n$-turn loop, multiply $R_r$ by $n^2$ and $h$ by $n$.

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## PROBLEMS

**2–6–1 Main-beam efficiency.** For an antenna with field pattern

$$E_n = \frac{\sin\theta}{\theta}\frac{\sin\phi}{\phi}$$

where $\theta =$ zenith angle (radians) and $\phi =$ azimuth angle (radians), (*a*) plot the normalized power pattern as a function of $\theta$; (*b*) using your graph, estimate the main beam efficiency of this antenna.

**2–7–1** **Directivity.** Show that the directivity $D$ of an antenna may be written

$$D = \frac{\dfrac{E(\theta,\phi)_{\max} E^*(\theta,\phi)_{\max}}{Z} r^2}{\dfrac{1}{4\pi} \displaystyle\iint_{4\pi} \dfrac{E(\theta,\phi) E^*(\theta,\phi)}{Z} r^2 \, d\Omega}$$

**2–7–2** **Approximate directivities.** Calculate the approximate directivity from the half-power beamwidths of a unidirectional antenna if the normalized power pattern is given by: (*a*) $P_n = \cos\theta$, (*b*) $P_n = \cos^2\theta$, (*c*) $P_n = \cos^3\theta$, and (*d*) $P_n = \cos^n\theta$. In all cases these patterns are unidirectional ($+z$ direction) with $P_n$ having a value only for zenith angles $0^\circ \le \theta \le 90^\circ$ and $P_n = 0$ for $0^\circ \le \theta \le 180^\circ$. The patterns are independent of the azimuth angle $\phi$.

***2–7–3** **Approximate directivities.** Calculate the approximate directivities from the half-power beamwidths of the three unidirectional antennas having power patterns as follows:

$$P(\theta,\phi) = P_m \sin\theta \sin^2\phi$$

$$P(\theta,\phi) = P_m \sin\theta \sin^3\phi$$

$$P(\theta,\phi) = P_m \sin^2\theta \sin^3\phi$$

$P(\theta,\phi)$ has a value only for $0 \le \theta \le \pi$ and $0 \le \phi \le \pi$ and is zero elsewhere.

***2–7–4** **Directivity and gain.** (*a*) Estimate the directivity of an antenna with $\theta_{HP} = 2^\circ$, $\phi_{HP} = 1^\circ$, and (*b*) find the gain of this antenna if efficiency $k = 0.5$.

**2–9–1** **Directivity and apertures.** Show that the directivity of an antenna may be expressed as

$$D = \frac{4\pi}{\lambda^2} \frac{\displaystyle\iint_{A_p} E(x,y)\,dx\,dy \iint_{A_p} E^*(x,y)\,dx\,dy}{\displaystyle\iint_{A_p} E(x,y)E^*(x,y)\,dx\,dy}$$

where $E(x,y)$ is the aperture field distribution.

**2–9–2** **Effective aperture and beam area.** What is the maximum effective aperture (approximately) for a beam antenna having half-power widths of 30° and 35° in perpendicular planes intersecting in the beam axis? Minor lobes are small and may be neglected.

***2–9–3** **Effective aperture and directivity.** What is the maximum effective aperture of a microwave antenna with a directivity of 900?

**2–11–1** **Received power and the Friis formula.** What is the maximum power received at a distance of 0.5 km over a free-space 1-GHz circuit consisting of a transmitting antenna with a 25-dB gain and a receiving antenna with a 20-dB gain? The gain is with respect to a lossless isotropic source. The transmitting antenna input is 150 W.

***2–11–2** **Spacecraft link over 100 Mm.** Two spacecraft are separated by 100 Mm. Each has an antenna with $D = 1000$ operating at 2.5 GHz. If craft $A$'s receiver requires 20 dB over 1 pW, what transmitter power is required on craft $B$ to achieve this signal level?

**2–11–3** **Spacecraft link over 3 Mm.** Two spacecraft are separated by 3 Mm. Each has an antenna with $D = 200$ operating at 2 GHz. If craft $A$'s receiver requires 20 dB over 1 pW, what transmitter power is required on craft $B$ to achieve this signal level?

**2–11–4** **Mars and Jupiter links.** (*a*) Design a two-way radio link to operate over earth-Mars distances for data and picture transmission with a Mars probe at 2.5 GHz with a 5-MHz bandwidth. A power of $10^{-19}$ W $\text{Hz}^{-1}$ is to be delivered to the earth receiver and $10^{-17}$ W $\text{Hz}^{-1}$ to the Mars receiver. The Mars antenna must be no larger than 3 m in diameter. Specify effective aperture of Mars and earth antennas and transmitter power (total over entire bandwidth) at each end. Take earth-Mars distance as 6 light-minutes. (*b*) Repeat (*a*) for an earth-Jupiter link. Take the earth-Jupiter distance as 40 light-minutes.

***2–11–5** **Moon link.** A radio link from the moon to the earth has a moon-based $5\lambda$-long right-handed monofilar axial-mode helical antenna (see Eq. (8–3–7)) and a 2-W transmitter operating at 1.5 GHz. What should the polarization state and effective aperture be for the earth-based antenna in order to deliver $10^{-14}$ W to the receiver? Take the earth-moon distance as 1.27 light-seconds.

**2–13–1** **Maximum phase error.** What is the phase difference between a point on the Fresnel-Fraunhofer boundary sphere and two points on the antenna, one, $R$, from the origin to the sphere perpendicular to the antenna shown in Fig. 2–17, and the other, $F$, from the antenna tip, $L/2$?

**2–15–1** **Semimajor and semiminor axes.** By rotating the coordinates of the polarization ellipse given in Eq. (2–15–7), (*a*) show that the tilt angle $\tau$ is given by

$$\tau = \frac{1}{2}\tan^{-1}\left(\frac{2E_1E_2\cos\delta}{E_1^2 - E_2^2}\right)$$

and (*b*) show that

$$OA = \left[(E_1\cos\tau + E_2\cos\delta\sin\tau)^2 + E_2^2\sin^2\delta\sin^2\tau\right]^{1/2} \quad \text{and}$$

$$OB = \left[(E_1\sin\tau + E_2\cos\delta\cos\tau)^2 - E_2^2\sin^2\delta\cos^2\tau\right]^{1/2}$$

**2–16–1** **Spaceship near moon.** A spaceship at lunar distance from the earth transmits 2-GHz waves. If a power of 10 W is radiated isotropically, find (*a*) the average Poynting vector at the earth, (*b*) the rms electric field **E** at the earth and (*c*) the time it takes for the radio waves to travel from the spaceship to the earth. (Take the earth-moon distance as 380 Mm.) (*d*) How many photons per unit area per second fall on the earth from the spaceship transmitter?

**2–16–2** **More power with CP.** Show that the average Poynting vector of a circularly polarized wave is twice that of a linearly polarized wave if the maximum electric field **E** is the same for both waves. This means that a medium can handle twice as much power before breakdown with circular polarization (CP) than with linear polarization (LP).

**2–16–3** **PV constant for CP.** Show that the instantaneous Poynting vector (PV) of a plane circularly polarized traveling wave is a constant.

***2–16–4** **EP wave power.** An elliptically polarized wave in a medium with constants $\sigma = 0$, $\mu_r = 2$, $\varepsilon_r = 5$ has $H$-field components (normal to the direction of propagation and normal to each other) of amplitudes 3 and 4 A $\text{m}^{-1}$. Find the average power conveyed through an area of 5 $\text{m}^2$ normal to the direction of propagation.

**2–17–1** **Crossed dipoles for CP and other states.** Two $\lambda/2$ dipoles are crossed at 90°. If the two dipoles are fed with equal currents, what is the polarization of the radiation perpendicular to the plane of the dipoles if the currents are (*a*) in phase, (*b*) phase quadrature (90° difference in phase), and (*c*) phase octature (45° difference in phase)?

***2–17–2 Polarization of two LP waves.** A wave traveling normally out of the page (toward the reader) has two linearly polarized components

$$E_x = 2\cos\omega t$$
$$E_y = 3\cos(\omega t + 90^\circ)$$

(*a*) What is the axial ratio of the resultant wave?
(*b*) What is the tilt angle $\tau$ of the major axis of the polarization ellipse?
(*c*) Does **E** rotate clockwise or counterclockwise?

**2–17–3 Superposition of two EP waves.** A wave traveling normally outward from the page (toward the reader) is the resultant of two elliptically polarized waves, one with components of **E** given by

$$E'_y = 2\cos\omega t$$
$$E'_x = 6\cos\left(\omega t + \tfrac{\pi}{2}\right)$$

and the other with components given by

$$E''_y = 1\cos\omega t$$
$$E''_x = 3\cos\left(\omega t - \tfrac{\pi}{2}\right)$$

(*a*) What is the axial ratio of the resultant wave?
(*b*) Does **E** rotate clockwise or counterclockwise?

***2–17–4 Two LP components.** An elliptically polarized plane wave traveling normally out of the page (toward the reader) has linearly polarized components $E_x$ and $E_y$. Given that $E_x = E_y = 1$ V m$^{-1}$ and that $E_y$ leads $E_x$ by 72°,

(*a*) Calculate and sketch the polarization ellipse.
(*b*) What is the axial ratio?
(*c*) What is the angle $\tau$ between the major axis and the $x$ axis?

**2–17–5 Two LP components and Poincaré sphere.** Answer the same questions as in Prob. 2–17–4 for the case where $E_y$ leads $E_x$ by 72° as before but $E_x = 2$ V m$^{-1}$ and $E_y = 1$ V m$^{-1}$.

***2–17–6 Two CP waves.** Two circularly polarized waves intersect at the origin. One ($y$ wave) is traveling in the positive $y$ direction with **E** rotating clockwise as observed from a point on the positive $y$ axis. The other ($x$ wave) is traveling in the positive $x$ direction with **E** rotating clockwise as observed from a point on the positive $x$ axis. At the origin, **E** for the $y$ wave is in the positive $z$ direction at the same instant that **E** for the $x$ wave is in the negative $z$ direction. What is the locus of the resultant **E** vector at the origin?

***2–17–7 CP waves.** A wave traveling normally out of the page is the resultant of two circularly polarized components $E_{\text{right}} = 5e^{j\omega t}$ and $E_{\text{left}} = 2e^{j(\omega t + 90^\circ)}$ (V m$^{-1}$). Find (*a*) the axial ratio AR, (*b*) the tilt angle $\tau$, and (*c*) the hand of rotation (left or right).

**2–17–8 EP wave.** A wave traveling normally out of the page (toward the reader) is the resultant of two linearly polarized components $E_x = 3\cos\omega t$ and $E_y = 2\cos(\omega t + 90^\circ)$. For the resultant wave find (*a*) the axial ratio AR, (*b*) the tilt angle $\tau$, and (c) the hand of rotation (left or right).

***2–17–9 CP waves.** Two circularly polarized waves traveling normally out of the page have fields given by $E_{\text{left}} = 2e^{-j\omega t}$ and $E_{\text{right}} = 3e^{j\omega t}$ (V m$^{-1}$) (rms). For the resultant wave find (*a*) AR, (*b*) the hand of rotation, and (*c*) the Poynting vector.

**2–17–10 EP waves.** A wave traveling normally out of the page is the resultant of two elliptically polarized (EP) waves, one with components $E_x = 5\cos\omega t$ and $E_y = 3\sin\omega t$ and another with components $E_r = 3e^{j\omega t}$ and $E_l = 4e^{-j\omega t}$. For the resultant wave, find (*a*) AR, (*b*) $\tau$, and (*c*) the hand of rotation.

***2–17–11 CP waves.** A wave traveling normally out of the page is the resultant of two circularly polarized components $E_r = 2e^{j\omega t}$ and $E_l = 4e^{-j(\omega t+45^\circ)}$. For the resultant wave, find (*a*) AR, (*b*) $\tau$, and (*c*) the hand of rotation.

**2–17–12 Circular-depolarization ratio.** If the axial ratio of a wave is AR, show that the circular-depolarization ratio of the wave is given by

$$R = \frac{AR-1}{AR+1}$$

Thus, for pure circular polarization $AR = 1$ and $R = 0$ (no depolarization) but for linear polarization $AR = \infty$ and $R = 1$.

For computer programs see Appendix C.